

# Fish Stock Assessment Manual

FAO  
FISHERIES  
TECHNICAL  
PAPER

**393**

by  
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***DANIDA***



**Rome, 2003**

## PREPARATION OF THIS DOCUMENT

The author, Emygdio Cadima, now retired, was an FAO scientist in the Fisheries Department until 1974, when he returned to the Instituto de Investigação das Pescas e do Mar (IPIMAR) in Portugal, having also been a Professor at the University of Algarve until 1997. At the end of 1997 he was the lecturer of a course in Fish Stock Assessment in IPIMAR, which became the basis for the preparation of this manual, requested and supported by the Project FAO/DANIDA GCP/INT/575/DEN. This manual also incorporates notes from courses in Fish Stock Assessment held at several different venues in the world, mainly in Europe, Latin America and Africa. These courses had an active collaboration of fisheries scientists from all over the world, especially Portugal. These scientists are also co-responsible for the orientation, for the matters treated and particularly for the elaboration of the exercises.

This manual aims to present the basic knowledge on the problems and methods of fish stock assessment to young scientists, post-graduate students, and PhD students. This is a scientific area in permanent development, where the knowledge of fisheries biology is applied in order to make a rational and sustained exploitation of the fishing resources.

The “Manual of Fish Stock Assessment” is mainly concerned with the theoretical aspects of the most used models for fish stock assessment. The practical application (i.e. the exercises solved in a spreadsheet), is considered as a complementary part to help the understanding of the theoretical matters.

The editing of the manuscript was made by Siebren Venema, manager of Project GCP/INT/575/DEN and Ana Maria Caramelo, Fishery Resources Officer in the FAO Fisheries Department.

### **Distribution:**

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Fisheries Education Institutes  
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Cadima, E.L.

Fish stock assessment manual.

*FAO Fisheries Technical Paper*. No. 393. Rome, FAO. 2003. 161p.

### **ABSTRACT**

The manual follows the same order of the lectures in the last course held in IPIMAR (November/December 1997). It starts with an introduction to the mathematical models applied in Fish Stock Assessment and some considerations on the importance of fisheries. The need for a rational management of the fishing resources is then stressed, this being indispensable for an adequate exploitation, aiming at conservation, to occur. The basic assumptions about a model and the concepts of different variation rates of a characteristic in relation to time (or to other characteristics) are presented, highlighting the most important aspects of the simple and exponential linear models which are used in the chapters that follow. After some considerations on the concept of cohort, models for the evolution in time of the number and weight of the individuals that constitute the cohort are developed, including models for the individual growth of the cohort. In the chapter concerning the study of the stock, the fishing pattern and its components are defined, the most used models for the stock–recruitment relation are presented, as well as the short and long term projections of a stock. With regard to fishing resources management, the discussion is focused on the biological reference points (target points, limit points and precautionary points) and fisheries regulation measures. The last chapter, which presents and discusses theoretical models of fish stock assessment, deals with production models (also designated as general production models) and with the long and short–term projections of the catches and biomasses. Finally, the general methods of estimating parameters are described and some of the most important methods are presented, with special relevance to the cohort analysis by age and length. Then a solution of the exercises from the last course held in IPIMAR, is presented by the author and the scientist Manuela Azevedo.

# TO MY FIRST MASTERS AND OLD-TIME FRIENDS

**Ray Beverton  
John Gulland  
Gunnar Sætersdal**

## PREFACE

This work is essentially orientated to present an introduction to the mathematical models applied to fisheries stock assessment.

There are several types of courses about the methods used in fish stock assessment.

One type considers practical application as the main aspect of the course, including the use of computer programs. The theoretical aspects are referred to and treated as complementary aspects.

A second type is mainly concerned with the theoretical aspects of the most used models. The practical application, considered as the complementary part, facilitates the understanding of the theoretical subjects.

In this work, the second type was adopted and exercises were prepared to be solved in a worksheet (Microsoft Excel). The table of contents indicates the exercises corresponding to each subject.

This manual is the result of a series of courses on Fish Stock Assessment held in the following places. Portugal : Instituto de Investigação das Pescas e do Mar – IPIMAR (ex-INIP) in Lisbon, Faculdade de Ciências de Lisboa, University of Algarve and Instituto de Ciências Biomédicas de Abel Salazar in Oporto. Other courses were held at Instituto de Investigação das Pescas in Cape Verde, at the Centro de Investigação Pesqueira in Angola, at the Instituto de Investigação das Pescas in Mozambique, at the Centro de Investigacion Pesquera – CIP in Cuba, at the Instituto del Mar del Perú – IMARPE in Peru, at the Instituto Español de Oceanografía – IEO (Vigo and Málaga – Spain). It is also a result of some lectures integrated into cooperation courses held in several countries and organized by FAO, by SIDA (Sweden), by NORAD (Norway) and by ICCAT.

Other fisheries scientists cooperated in these courses and they are also co-responsible for the orientation of the subjects studied and very particularly for the elaboration of the exercises and the editorial work. With no particular criterium, these are some of the collaborators to whom I express my appreciation: Ana Maria Caramelo, Manuel Afonso Dias, Pedro Conte de Barros, Manuela Azevedo Lebre, Raúl Coyula, Renato Guevara.

Lisbon, December 1997  
E. Cadima

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# GLOSSARY OF TECHNICAL TERMS USED IN THE MANUAL

**Abundance index (U)** – A characteristic preferably proportional to the available biomass of the resource. The catch per unit effort, **cpue** (especially when the effort is expressed in appropriate units) is an important index..

**Biological Limit Reference Point (LRP)** – Biological reference point indicating *limits* of the fishery exploitation with regard to stock self-reproduction, aiming at conservation of the resource.

**Biological Precautionary Reference Point (PaRP)** – biomass levels (**Bpa**) and fishing levels (**Fpa**), established under the precautionary principle, concerning the reproduction of the stock, aiming at conservation of the resources. The assumptions and methods used to determine the PaRPs should be mentioned.

**Biological Reference Point (BRP)** – Values of **F** and **B**, taking into consideration the *best possible catch* and/or ensuring the *conservation* of the fishery resource. There are BRPs based on long term projections (LP), BRPs based on values observed during a certain period of years and BRPs based on the two previous criteria. The BRPs can be *Target-Points (TRP)*, *Limit-Points (LRP)*, and *Precautionary Points (PaRP)*. In this manual the following biological reference points are referred to: **F<sub>max</sub>**, **F<sub>0.1</sub>**, **F<sub>high</sub>**, **F<sub>med</sub>**, **F<sub>MSY</sub>**, **F<sub>loss</sub>**, **F<sub>crash</sub>**, **B<sub>max</sub>**, **B<sub>0.1</sub>**, **B<sub>med</sub>**, **B<sub>MSY</sub>**, **B<sub>loss</sub>**, **MBAL**. Other biological reference points, used in management, like **F<sub>30%SPR</sub>**, are not mentioned in this manual.

**Biological Target Reference Point (TRP)** – Biological reference point indicating long term *objectives* (or targets), for the management of a fishery, taking into consideration the *best possible catch* and ensuring the conservation of the stock.

**Biomass (B)** – Weight of an individual or a group of individuals contemporaneous of a stock.

**Capturability Coefficient (q)** – Fraction of the biomass that is caught by unit of fishing effort.

**Carrying capacity (k)** – Capacity of the environment to maintain the stock living in it. It is, theoretically, the limit of the non exploited biomass (see intrinsic gross rate of the biomass, **r**).

**Catch in number (C)** – Number of individuals caught.

**Catch in weight or Yield (Y)** – Biomass of the stock taken by fishing. Yield does not necessarily correspond to *landed weight*. The difference between the two values, yield and landings, is mainly due to *rejections to the sea* of part of the catch which, for some reason (price, quality, space problems or even legal reasons), is not landed.

**Cohort** – Set of individuals of a fishery resource born from the same spawning.

**Exploitation pattern of a gear (s)** – Fraction of the individuals of a given size, available to the gear, which is caught. Also designated by *Selectivity* or *partial recruitment*.

**Individual growth coefficient (K)** – Instantaneous relative rate of change of a function of the individual weight, **w**, that is, **H(w<sub>∞</sub>)-H(w)**, where **w<sub>∞</sub>** is the *asymptotic individual weight* and **H(w)** is a function of **w** (frequently a power function, including the logarithmic function). The adopted models for the function **H(w)** have two constants, **w<sub>∞</sub>** and **K**. Some models introduce one more parameter, **b**, which is used to obtain a general relation



to include the most common individual growth relations. The constant **K** has the physical dimension of time<sup>-1</sup>.

**Individual Quota (IQ)** – Quota attributed to a vessel.

**Individual Transferable Quotas (ITQ)** – System of fisheries management characterized by the sale, at auctions, of the fishing annual vessel quotas.

**Minimum Biomass Acceptable Level (MBAL)** – Biological reference limit point that indicates a spawning biomass level under which the observed biomasses during a period of years, are small and the associated recruitments are smaller than the mean or median recruitment.

**Number of individuals of a cohort or of a stock (N)** – Number of survivals of a cohort (or a stock) at a certain instant or over an interval of time.

**Partial recruitment** – (see exploitation pattern)

**Precautionary principle** – This principle establishes that a lack of information does not justify the absence of management measures. On the contrary, management measures should be established in order to maintain the conservation of the resources. The assumptions and methods used for the determination of the scientific basis of the management should be presented.

**Production models** – Models that consider the biomass of the stock as a whole, that is, they do not take into consideration the age or size structure of the stock. These models are only applied in analyses that consider fishing level changes, as they do not allow the analysis of the effects of changes in the exploitation pattern, on catches and biomasses.

**Quota (Q)** – Each of the fractions in which the TAC was divided.

## ***RATES***

**Absolute Instantaneous Rate of y, air(y)** – Velocity of the variation of the function **y(x)**, at the instant **x**.

**Absolute Mean Rate of y, amr(y)** – Mean velocity of the variation of the function **y(x)**, during a certain interval of **x**.

**Annual Survival Rate (S)** – Mean rate of survivals of a cohort during one year, relative to the initial number.

**Exploitation Rate (E)** – Ratio between the number of individuals caught and the total number of individuals dead, over a certain period of time, that is, **E = C/D**.

**Fishing mortality instantaneous rate (F) (Fishing mortality coefficient)** – Relative instantaneous rate of the mortality of the number of individuals that die due to fishing .

**Intrinsic rate of the biomass growth (r)** – Constant of the *Production models* that represents the instantaneous rate of the decreasing of the function **H(K)-H(B)**, where **B** is the biomass, **H(B)** is a function of the total biomass, usually a power-function, (including the logarithmic function that can be considered a limit power function) and **k** is the **carrying capacity** of the environment. Some models introduce one more parameter, **p**, which is used to obtain a more general relation.

**Natural mortality instantaneous rate (M)** (Natural mortality Coefficient) – Instantaneous relative rate of the mortality of the number of individuals that die due to all causes other than fishing.

**Relative instantaneous rate of y,  $\text{rir}(y)$**  – Velocity of the variation of the function  $y(x)$ , relative to the value of  $y$ , at the instant  $x$ .

**Relative mean rate of y,  $\text{rmr}(y)$**  – Mean velocity of the variation of the function  $y(x)$  relative to a value of  $y$ , during a certain interval of  $x$ .

**Total mortality instantaneous rate (Z) (Total mortality coefficient)** – Relative instantaneous rate of the mortality of the number of individuals that die due to all causes.  $Z$ ,  $F$  and  $M$  are related by the following expression :  $Z=F+M$  .

**Recruitment to the exploitable phase (R)** – Number of individuals of a stock that enter the fishery area for the first time each year.

**Selectivity** – (see exploitation pattern)

**Spawning or adult biomass (SP)** – Biomass of the stock (or of a cohort) which has already spawned at least once.

**Stock** – Set of survivals of the cohorts of a fishery resource, at a certain instant or period of time. It may concern the biomass or the number of individuals.

**Stock-Recruitment (S-R) relation** – Relation between the parental stock ( spawning biomass) and the resulting recruitment (usually the number of recruits to the exploitable phase). The models have two constants,  $\alpha$  and  $k$ . The constant  $k$  has the physical dimension of weight and  $\alpha$  has the dimension of  $\text{weight}^{-1}$ . Some models introduce one more parameter,  $c$ , which is used to obtain a general relation that includes the most common relations.

**Structural models** – Models that consider the structure of the stock by ages or sizes. These models allow one to analyse the effects on catches and biomasses, due to changes in the fishing level and exploitation pattern.

**Total Allowable Catch (TAC)** – Management measure that limits the total annual catch of a fishery resource, aiming to indirectly limit the fishing mortality. The TAC can be divided into *Quotas (Q)* using different criteria, like countries, regions, fleets or vessels.

**Total number of deaths (D)** – Total number of individuals that die during a certain period of time..

**Virgin biomass (VB)** – Biomass of the stock not yet exploited.

## SYMBOLS

<b>Symbols</b>	<b>Indicating :</b>
<b>A</b>	Constant of the simple linear model (intercept of the straight line )
$\alpha$	Constant of the Stock-Recruitment relations (limit value of R/S when $S \rightarrow 0$ )
<b>amr(y)</b>	Absolute mean rate of variation of y
<b>air(y)</b>	Absolute instantaneous rate of variation of y
<b>B</b>	Constant of the simple linear model (slope of the straight line)
<b>B</b>	Biomass
<b>SP, SB</b>	Spawning Biomass
<b>C</b>	Catch, in number
<b>C</b>	Constant of the Stock-Recruitment relations (generalizes the models)
<b>D</b>	Total number of deaths
<b>E</b>	Exploitation rate
<b>F</b>	Fishing mortality coefficient
<b>Cconst</b>	Non defined constant
<b>Cte</b>	Non defined constant
<b>H</b>	General power function
<b>ITQ</b>	Individual Transferable Quotas
<b>K</b>	Constant of the individual growth models (associated to growth rate)
<b>k</b>	Constant of the Stock-Recruitment relations
<b>k</b>	Constant of the production models (Carrying capacity)
<b>L,L</b>	Total length of an individual
<b>MBAL</b>	Minimum Biomass Acceptable Level (biological reference limit point)
<b>M</b>	Natural mortality coefficient
<b>N</b>	Number of individuals of a cohort
<b>P</b>	Constant of the Production models (generalizes the models)
<b>Q</b>	Capturability coefficient
<b>R</b>	Constant of the Production models (intrinsic rate associated with the biomass growth)
<b>R<sup>2</sup></b>	Determination coefficient
<b>R</b>	Recruitment to the exploitable phase
<b>rmr(y)</b>	Relative mean rate of variation of y
<b>rir(y)</b>	Relative instantaneous rate of variation of y

<b>Symbols</b>	<b>Indicating :</b>
<b>S</b>	Annual Survival rate
<b>S</b>	Adult or total biomass (in the relations S-R)
<b>s</b>	Exploitation pattern (selectivity)
<b>SQ</b>	Sum of the squares of the deviations
<b>S-R</b>	Stock-Recruitment relation
<b>t</b>	Instant of time
<b>T</b>	Interval of time between 2 instants
<b>TAC</b>	Total Allowed Catch
<b>TRP</b>	Biological Target Reference Point
<b>U</b>	Stock abundance index
<b>V</b>	Function to be maximized for the determination of $F_{0.1}$
<b>W</b>	Individual weight
<b>Y</b>	Catch in weight
<b>Z</b>	Total mortality coefficient (total mortality instantaneous rate)

## SUBSCRIPTS

The characteristics of this glossary are usually shown with indices; that is why it was considered necessary to present the meaning of those subscripts.

### **Subscripts      Indicating :**

<b>\$</b>	Economical value of the respective characteristic of the cohort
<b><math>\lambda</math></b>	Maximum age
<b>0.1</b>	Value of F (and of other characteristics of the cohort) corresponding to the age of the biomass equal to 10 percent of the virgin biomass
<b>c</b>	Recruitment to exploitable phase
<b>crash</b>	Value of F which, at long term, corresponds to the collapse value of the spawning biomass
<b>E</b>	Value of the characteristics of the cohort corresponding to an equilibrium point
<b>i</b>	Age
<b>infl</b>	Value of the characteristic corresponding to an inflection point of any relation between that characteristic and other variable.
<b>l</b>	Length
<b>lim</b>	Value of B or of F corresponding to a biological reference limit point
<b>loss</b>	Value of B or of F corresponding to the minimum spawning biomass observed
<b>Max</b>	Value of F (and of other characteristics of the cohort) where the yield per recruit is maximum
<b>Med</b>	Value of F (and of other characteristics of the cohort) which, at long term, will produce a spawning biomass per recruit equal to the median value of the spawning biomasses per recruit observed during a certain period of years
<b>MSY</b>	Value of F (and of other characteristics of the stock) where the long term total yield is maximum
<b>R</b>	Recruitment to the exploitable phase

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# CHAPTER 1 – INTRODUCTION

## 1.1 THE IMPORTANCE OF FISHERIES

The importance of fisheries in a country cannot only be measured by the contribution to the GDP, but one must also take into consideration that fisheries resources and products are fundamental components of human feeding and employment.

Another aspect that makes fisheries resources important is the self renewable character. Unlike mineral resources, if the fishery resources or any other biological resources are well managed, their duration is practically unlimited.

An important conclusion is that the fundamental basis for the conservation and management of fisheries resources stems from the biological characteristics. (This does not mean that social, economic or any other effects are not important for management).

In Portugal, the fisheries contribution to the GDP is less than 1.5 percent. However, with regard to food, the annual consumption value of 60 kg of fish per person, is very high. Only countries like Iceland, Japan and some small insular nations reach a higher value. We still have to consider that of the total amount of protein necessary in our food consumption, 40 percent comes from fisheries. This corresponds to 15 percent of the total amount spent on food by the Portuguese population.

From a social point of view, we estimate that there are, at present, 34 000 fishermen in Portugal. Assuming that each job at sea generates 4 or 5 jobs on land (canning, freezing and fish meal industry, commercialization, administration, research and training, etc.) one can estimate that about 150 000 Portuguese work in the several sectors of fisheries. Consequently, taking a minimum of 3 people per family, it is not unreasonable to say that about half a million Portuguese people depend on fisheries activities for their livelihoods.

## 1.2 FISHERIES RESOURCES MANAGEMENT

Sætersdal (1984) defined a general principle of fisheries management as :

**“to obtain the BEST POSSIBLE utilization of the resource  
for the benefit of the COMMUNITY”**

It will be necessary to define, in each particular case, what best, possible and community mean.

In fact, best can be taken as :

- Bigger yield
- Bigger value of the catch
- Bigger profit (difference between the value of the landing and the costs of exploitation)
- More foreign currency
- More jobs, etc.

Community may also be taken as :

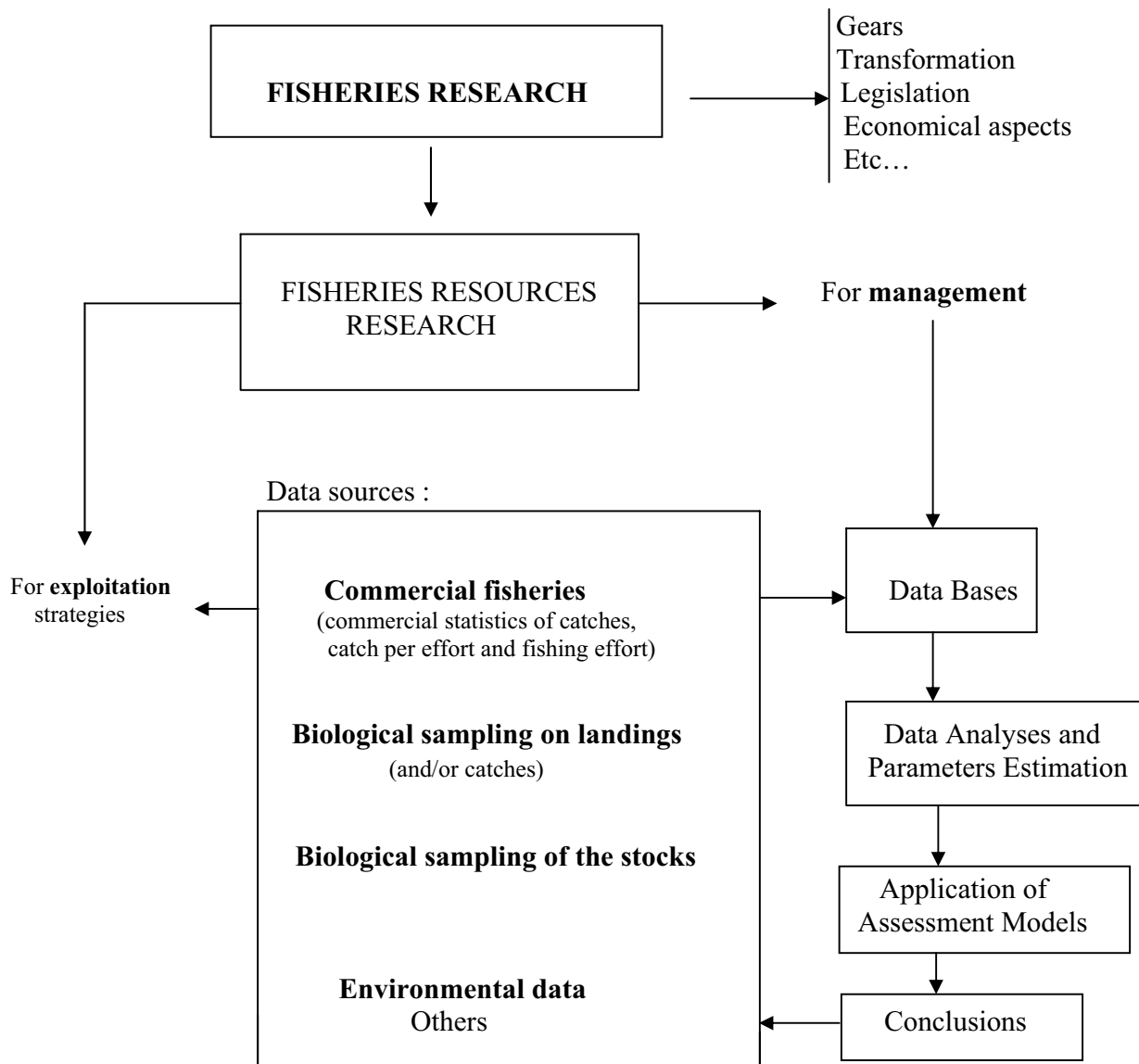
- The population of the world
- The European Community
- A country
- A region
- Groups of interests (fishermen, shipowners, consumers, ...)

Possible

reminds us that we cannot forget the self renewable character of fisheries resources and consequently, that the conservation of the fishery resource must be guaranteed in order to allow the application of the general principle for a long period of time. This statement means that conservation of an ecosystem does not imply that one should attribute the same importance to all its components.

### **1.3 FISHERIES RESOURCES RESEARCH**

Figure 1.1 shows that the research on fisheries resources covers several sectors of the fishing activity. The assessment models are the main concern of this manual. Among the several works and books on fish stock assessment, the books and/or manuals by Beverton & Holt (1956), Ricker (1958, 1975) and Gulland (1969, 1983) are historical standing references.



**Figure 1.1 The several sectors of fishing activity**

#### **1.4 FISH STOCK ASSESSMENT**

The following are necessary to assess a fish stock:

- The appropriate data bases
- Analyses of the available data

- Short and long-term projections of the yield and biomass
- To determine long-term biological reference points
- To estimate the short and long-term effects on yield and biomass of different strategies of the fishery exploitation

The different steps to assess a stock can be summarized as follows :

- a) To define the objectives of the assessment according to the development phase of the fisheries and the available information.
  
- b) To promote the collection of information :
  - Fisheries commercial statistics : total and by resource landings, catch per effort, fishing effort (number of trips, days, tows, time spent fishing, etc.), and characteristics of the gears used.
  - Types of operation of the fleets and of its fishing gears, etc.
  - Biological sampling in the landing ports.
  - Biological sampling (and information about the fishing operation) on board commercial vessels.
  - Biological sampling on board research vessels.
  
- c) To analyse the stocks

The knowledge gained about the resource and the available basic data, determine the type of models that should be used and consequently the type of analyses that can be done. As an illustration, let us look at some general situations :

Fishery resource with little information



Analyses using particular methods to estimate biomasses and potential yields.

Fishery resource with data on catches and catch per effort (CPUE) or stock abundance indices during several years



Analyses using production models in order to make projections of yield and catch per effort.

Fishery resource with information collected over several years on :

- Biological distribution of the catches by species, by length, by ages, etc.
- Commercial catches
- Fishing effort or CPUE
- Research cruises (distribution of the stock by areas, by length, by ages, etc.)



Historical analysis of the stock (VPA)  
Long and short-term projections with different conditions (scenarios)

### ***Comments***

1. The lack of information may prevent certain projections, but allows other types of analyses.
2. The Precautionary Principle forces one to estimate and to project catches and biomasses, even if they are not very precise. This will be discussed later.

# CHAPTER 2 – MODELS AND RATES

## 2.1 MODELS

Science builds models or theories to explain phenomena. One observes *phenomena* and then looks for relations, causes and effects. Observations are made about the evolution of a magnitude (characteristics) with time (or with other characteristics) and possible causes (factors) are explored. Examples:

- Physics – *phenomenum of the movement of the bodies* (characteristics – distance related to time spent)
- Biology – *phenomenum of growth* (characteristic – length or weight, related to time).

### 2.1.1 STRUCTURE OF A MODEL

#### ***Basic assumptions***

The assumptions to serve as a basis for a model should :

- simplify reality
- be simple and mathematically treatable (manageable)
- not be contradictory
- not be demonstrated
- be established with the characteristics

Usually basic assumptions are related to the evolution of the characteristics. So, they are established on the variation rate of those characteristics and they do not need to be proved.

#### ***Relations (properties)***

- they are deduced from the basic assumptions by the laws of logic (mathematics). The properties are also designated by

**“results” or “conclusions” of the model.**

#### ***Verification***

- the results of the model must be coherent (to agree) with reality.

**This implies the application of statistical methods and sampling techniques to check the agreement of the results with the observations.**

### ***Improvement***

- if agreement is approximate, it is necessary to see if the approximation is enough or not.
- if the results do not agree with reality, then the basic assumptions have to be changed
- the changes can aim to the application of the model to other cases.

### ***Advantages***

- it is easier to analyse the properties of the model than the reality.
- the models produce useful results.
- they allow analysis of different situations or scenarios by changing values of the factors.
- to point out the essential parts of the phenomenon and its causes.
- they can be improved in order to adjust better to the reality.

## **2.1.2 SOME TYPES OF MODELS USED IN STOCK ASSESSMENT**

### ***Production Models***

The production models are also designated as General Production models, Global models, Synthetical models or Lotka-Volterra type models. These models consider the stock globally, in particular the total abundance (in weight or in number) and study its evolution, the relation with the fishing effort, etc.. They do not consider the structure of the stock by age or by size.

### ***Structural Models***

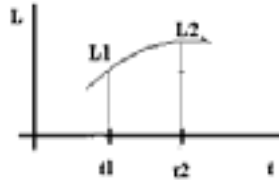
These models consider the structure of the stock by age and the evolution of the structure with time. They mainly recognize that the stock is composed of individuals of different cohorts, and, consequently, of different ages and sizes. So, they analyse and they project the stock and the catches for the coming years, by following the evolution of its different cohorts.

This manual will not follow the chronological construction of the models. It was thought to be more convenient to deal firstly with the structural models and afterwards with the production models.



## 2.2 RATES

The basic assumptions of a model, for the evolution of a characteristic, require the concept of variation rate of the characteristic related to time (or to other characteristics).

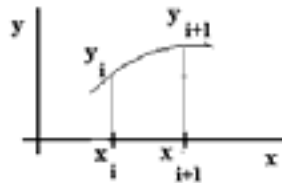


**Figure 2.1 Evolution of the length (L) of an individual with time (or age) (t)**

In order to generalize the study of the rates, the characteristic L in the example above will be substituted by y, and the associated variable will not be time, t, but the variable x. To study the stock assessment models and to make this study easier, it will be considered that the function y will only assume real and positive values.

### 2.2.1 ABSOLUTE MEAN RATE – amr (y)

Consider y a function of x and the interval i with the limits  $(x_i, x_{i+1})$



**Figure 2.2 Function  $y = f(x)$  with variation in the interval i**

Let :

$\Delta x_i = x_{i+1} - x_i$  be the size of the interval

$y_i$  = the value of y when  $x = x_i$

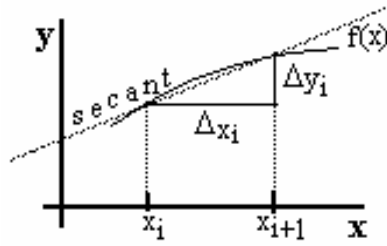
$y_{i+1}$  = the value of y when  $x = x_{i+1}$

The variation of y in the interval  $\Delta x_i$  will be  $\Delta y_i = y_{i+1} - y_i$

The absolute mean rate, amr (y), of the variation of y within the interval  $\Delta x_i$  will be :

$$\text{amr}(y) = \frac{\Delta y_i}{\Delta x_i}$$

Graphically :



**Figure 2.3 Absolute mean rate of the variation of y within the interval  $\Delta x_i$**

Slope of the secant =  $\frac{\Delta y_i}{\Delta x_i} = \text{amr (y) during } \Delta x_i$

Note: amr (y) is known in physics as the mean velocity of the variation of y with x, in the interval  $\Delta x_i$ .

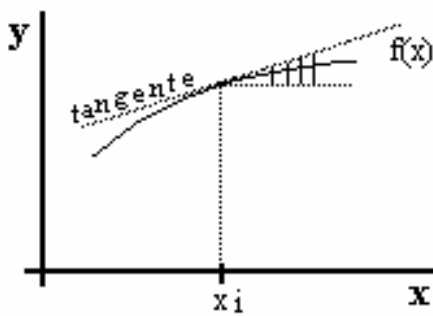
**2.2.2 ABSOLUTE INSTANTANEOUS RATE – air (y)**

Let y be a function of x

The absolute instantaneous rate of y at the point  $x = x_i$  is the derivative of y in order to x at that point.

$$\text{air}(y)_{x=x_i} = \left. \frac{dy}{dx} \right|_{x=x_i}$$

Graphically :



$\text{air}(y)_{x=x_i}$  equal to the slope of the tangent to the curve at the point  $x=x_i$

**Figure 2.4 Absolute instantaneous rate of y at point  $x_i$**

Note: air(y) is known as the instantaneous velocity of variation of y with x at the point x.

## ***Properties***

1. Given the value of  $\text{air}(y)$  the calculation of the function  $y$  is obtained, by integration, being  $y = f(x) + \text{Constant}$ , where  $f(x) = \text{Primitive of air}(y)$  and Constant is the constant of integration.

If the initial condition  $x^*, y^*$  is adopted, where  $y^*$  is the value of  $y$  corresponding to  $x = x^*$ , eliminating the Constant, then one can write  $y = y^* + f(x) - f(x^*)$

2. The angle made by the tangent to the curve  $y$  with the  $xx$ 's axis is designated by inclination.

The trigonometric tangent of the inclination is the slope of the geometrical tangent.

$$\text{air}(y) = \text{derivative of } y = \text{slope} = \text{tg}(\text{inclination})$$

3. If, at point  $x$  :

$\text{air}(y) > 0$  then  $y$  is increasing at that point

$\text{air}(y) < 0$  then  $y$  is decreasing at that point

$\text{air}(y) = 0$  then  $y$  is stationary at that point (maximum or minimum)

4. If  $\text{air}(y)$  is constant (= const) then  $y$  is a linear function. From property 1, it will be :

$$y = \text{Constant} + \text{const. } x \quad \text{or}$$

$$y = y^* + \text{const.}(x - x^*) \quad \text{and vice versa}$$

5. If  $y(x) = u(x) + v(x)$  then  $\text{air}(y) = \text{air}(u) + \text{air}(v)$

6. If factors A and B cause variations in  $y$ , then factors A and B considered *simultaneously* cause a variation of  $y$  with:

$$\text{air}(y)_{\text{total}} = \text{air}(y)_{\text{causeA}} + \text{air}(y)_{\text{causeB}}$$

$$\text{air}(\text{air}(y)) = \frac{d^2 y}{d^2 x} = \text{acceleration of } y \text{ at the point } x$$

7. If the acceleration at the point  $x$  is increasing, then  $\text{air}(y)$  is positive and if that acceleration is decreasing, then  $\text{air}(y)$  will be negative.

### **2.2.3 RELATIVE MEAN RATE - $\text{rmr}(y)$**

Consider  $y$  a function of  $x$  and the interval  $(x_i, x_{i+1})$

Let :

$$\Delta x_i = x_{i+1} - x_i = \text{the size of the interval}$$

$$y_i = \text{value of } y \text{ when } x = x_i$$

$y_{i+1}$  = value of  $y$  when  $x = x_{i+1}$

$x_i^*$  = a certain point in the interval  $(x_i, x_{i+1})$

$y_i^*$  = value of  $y$  when  $x = x_i^*$

$x_i^*$  can be  $x_i$ ,  $x_{i+1}$ ,  $x_{\text{central}_i}$ , etc.

The mean rate of  $y$  relative to  $y_i^*$  will be :

$$\text{rmr}(y) = \frac{1}{y_i^*} \cdot \frac{\Delta y_i}{\Delta x_i} \quad \text{or}$$

$$\text{rmr}(y) = \frac{1}{y_i^*} \cdot \text{amr}(y)$$

### Comments

1.  $\text{rmr}(y)$  is associated with the mean rate of the variation of the percentage of  $y$  in relation to  $y^*$ , that is:

$$\frac{\Delta\left(\frac{y_i}{y_i^*}\right)}{\Delta x_i}$$

2. Let  $x_{\text{central}_i}$  be  $x_{\text{central}_i} = x_i + \frac{\Delta x_i}{2} = \frac{1}{2} \cdot (x_i + x_{i+1}) = \bar{x}_i$

3. It is convenient to designate by  $y_{\text{central}_i}$  the value of  $y$  in the interval  $(x_i, x_{i+1})$  when  $x = x_{\text{central}_i}$ .

Notice that  $y_{\text{central}_i}$  can be different from the mean,  $\frac{(y_i + y_{i+1})}{2}$

4. It is frequent to calculate  $\text{rmr}(y)$  in relation to  $y_{\text{central}_i}$  of the interval.

## 2.2.4 RELATIVE INSTANTANEOUS RATE - $\text{rir}(y)$

Let  $y$  be a function of  $x$ .

The relative instantaneous rate of  $y$  at the point  $x = x_i$  is

$$\text{rir}(y) = \frac{1}{y_i} \cdot \left. \frac{dy}{dx} \right|_{x=x_i} \quad \text{or} \quad \text{rir}(y) = \frac{\text{air}(y)_{x=x_i}}{y_i}$$

## Properties

1. Given  $\text{rir}(y)$ , the calculation of the function  $y$  is obtained by integration, being

$y = f(x) + \text{Constant}$ , where  $f(x) = \text{Primitive of } \text{rir}(y)$  and  $C$  is the constant of integration.

If one accepts the initial condition  $x^*$ ,  $y^*$ , where  $y^*$  is the value of  $y$  corresponding to  $x = x^*$ , one will get, eliminating the Constant,  $y = y^* + f(x) - f(x^*)$

2. If, at a point  $x$  :

$\text{rir}(y) > 0$       then  $y$  is increasing at that point  
 $\text{rir}(y) < 0$       then  $y$  is decreasing at that point  
 $\text{rir}(y) = 0$       then  $y$  is stationary at that point (maximum or minimum)

3.  $\text{rir}(y) = \text{air}(\ln y)$  as can be deduced from the derivation rules.

4. If  $\text{rir}(y) = \text{constant} = (\text{const})$  then  $y$  is an exponential function of  $x$ , that is,

$$y = \text{Constant} \cdot e^{\text{const} \cdot x} \quad \text{or}$$

$$y = y^* \cdot e^{\text{const} \cdot (x - x^*)} \quad \text{and vice-versa}$$

5. If  $y(x) = u(x) \cdot v(x)$  then  $\text{rir}(y) = \text{rir}(u) + \text{rir}(v)$

6. If the factors  $A$  and  $B$  cause variations in  $y$ , then simultaneously, factors  $A$  and  $B$  cause a variation in  $y$ , with:

$$\text{rir}(y)_{\text{total}} = \text{rir}(y)_{\text{cause A}} + \text{rir}(y)_{\text{cause B}}$$

## 2.3 SIMPLE LINEAR MODEL

Let  $y = f(x)$

### Basic assumption of the model

$\text{air}(y) = \text{Constant} = b$  in the interval  $(x_i, x_{i+1})$  with  $\Delta x_i = x_{i+1} - x_i$

### Initial Condition

$$x^* = x_i \quad \heartsuit \quad y^* = y_i$$

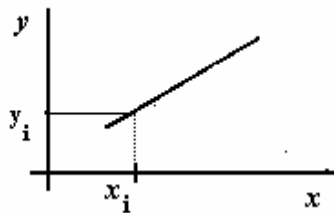


Figure 2.5 Graphical representation of a simple linear model

### ***Properties***

1. General expression  $y = y_i + b \cdot (x - x_i) ; y = a + bx$
2. Value  $y_{i+1}$  at the end of the interval,  $\Delta x_i$   $y_{i+1} = y_i + b \cdot \Delta x_i$
3. Variation,  $\Delta y_i$ , in the interval,  $\Delta x_i$   $\Delta y_i = y_{i+1} - y_i = b \cdot \Delta x_i$
4. Central value,  $y_{\text{central}_i}$  of the interval,  $\Delta x_i$   $y_{\text{central}_i} = y_i + b \cdot (x_{\text{central}_i} - x_i) = y_i + b \cdot \frac{\Delta x_i}{2}$
5. Cumulative value,  $y_{\text{cum}_i}$  during the interval,  $\Delta x_i$   $y_{\text{cum}_i} = \int_{x_i}^{x_{i+1}} y \cdot dx = \Delta x_i \cdot (a + b \cdot \bar{x}_i)$   
or from the Property 1  
 $y_{\text{cum}_i} = \Delta x_i \cdot [y_i + b \cdot (\bar{x}_i - x_i)]$
6. Mean value,  $\bar{y}_i$ , in the interval,  $\Delta x_i$   $\bar{y}_i = \frac{y_{\text{cum}_i}}{\Delta x_i} = a + b \cdot \bar{x}_i$  where  
 $\bar{y}_i = \frac{y_{\text{cum}_i}}{\Delta x_i} = y_i + b \cdot (\bar{x}_i - x_i)$

### ***Other useful expressions***

7. Cumulative value,  $y_{\text{cum}_i}$  during the interval,  $\Delta x_i$   $y_{\text{cum}_i} = \Delta x_i \cdot \bar{y}_i$
8. Mean value,  $\bar{y}_i$ , during the interval,  $\Delta x_i$   $\bar{y}_i = y_i + b \cdot (\bar{x}_i - x_i)$  where  $\bar{y}_i = a + b \bar{x}_i$
9. Mean value,  $\bar{y}_i$ , in the interval,  $\Delta x_i$   $\bar{y}_i = y_i + b \frac{\Delta x_i}{2}$
10. Mean value,  $\bar{y}_i$ , during the interval,  $\Delta x_i$   $\bar{y}_i = y_{\text{central}_i}$
11. Relation between  $\text{amr}(y)$  et  $\text{air}(y)$   $\text{amr}(y_i) = \frac{\Delta y_i}{\Delta x_i} = b = \text{air}(y)$

12. If  $\Delta y_i < 0$  then  $b < 0$  et vice-versa
13. In the linear model, the arithmetic mean of  $y_i$  and  $y_{i+1}$  is equal to the mean value,  $\bar{y}_i$ , and equal to the central value  $y_{\text{central}_i}$

### ***Important demonstrations***

General expression  
Property 1      If  $tia(y) = b$  in the interval  $\Delta x_i$  then  $y$  is linear with  $x$  and considering the initial condition it will be:  $y = y_i + b.(x-x_i)$

Central value  
Property 4       $y_{\text{central}_i} = y_i + b.(x_{\text{central}_i} - x_{\text{central}_i}) = y_i + b \left( \frac{x_{i+1} + x_i}{2} - x_i \right) = y_i + b \cdot \frac{\Delta x_i}{2}$

Cumulative value  
Property 5      from the definition of the cumulative value :

$$y_{\text{cum}_i} = \int_{x_i}^{x_{i+1}} (a + bx) \cdot dx$$

$$= a(x_{i+1} - x_i) + b \left( \frac{x_{i+1}^2}{2} - \frac{x_i^2}{2} \right)$$

it will be necessary to use the factorization of the difference of two squares, that is :

$$x_{i+1}^2 - x_i^2 = (x_{i+1} - x_i).(x_{i+1} + x_i) = \Delta x_i \cdot (x_{i+1} + x_i)$$

and then:

$$y_{\text{cum}_i} = a\Delta x_i + b\Delta x_i \cdot \bar{x}_i = \Delta x_i (a + b \cdot \bar{x}_i)$$

$\bar{y}_i$  et  $y_{\text{central}_i}$   
Property 10       $\bar{y}_i = y_i + b \left( \bar{x}_i - x_i \right) = y_i + b \left( \frac{x_{i+1} + x_i}{2} - x_i \right) = y_i + b \cdot \frac{\Delta x_i}{2} = y_{\text{central}_i}$

## **2.4 EXPONENTIAL MODEL**

Let  $y = f(x)$

### ***Basic assumption of the model***

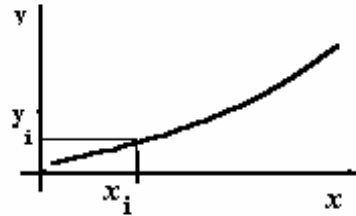
$rir(y) = \text{Constant} = c$  in the interval  $(x_i, x_{i+1})$ , with  $\Delta x_i = x_{i+1} - x_i$

### ***Initial condition***

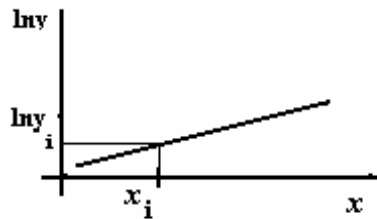
$$x^* = x_i \quad \heartsuit \quad y^* = y_i$$

**Properties**

$\text{rir}(y) = \text{air}(\ln y)$  means that the exponential model of  $y$  against  $x$  is equivalent to the linear model of  $\ln y$  against  $x$ . So being, its properties can be deduced by backwards application of logarithm rules to the properties of the linear model of  $\ln y$  against  $x$ .



**Figure 2.6** Graphical representation of the exponential model



**Figure 2.7** Graphical representation of the linear model of  $\ln y$  against  $x$

	Exponential model of $y$ ( $y$ against $x$ )	Linear model $\ln y$ ( $\ln y$ against $x$ )
1. General expression	$y = y_i \cdot e^{c(x-x_i)}$	$\ln y = \ln y_i + c(x-x_i)$
2. Value of $y_{i+1}$ at the end of the interval, $\Delta x_i$	$y_{i+1} = y_i \cdot e^{c \Delta x_i}$	$\ln y_{i+1} = \ln y_i + c \Delta x_i$
3. Variation, $\Delta y_i$ , during the interval, $\Delta x_i$	$\Delta y_i = y_{i+1} - y_i = y_i \cdot (e^{c \Delta x_i} - 1)$ calculated from 1	
4. Central value, $y_{\text{central}_i}$ , in the interval $\Delta x_i$	$y_{\text{central}_i} = y_i \cdot e^{\frac{c \Delta x_i}{2}}$	$\ln y_{\text{central}_i} = \ln y_i + \frac{c \Delta x_i}{2}$
	$y_{\text{central}_i} = (y_i \cdot y_{i+1})^{1/2}$	$\ln y_{\text{central}_i} = (\ln y_i + \ln y_{i+1})/2$
	$(y_{\text{central}_i} = \text{geometric mean of the extremes } y_i \text{ and } y_{i+1})$	



5. Cumulative value,  $y_{cum_i}$ , during the interval,  $\Delta x_i$  
$$y_{cum_i} = \int_{x_i}^{x_{i+1}} y \cdot dx = \frac{\Delta y_i}{c}$$
6. Mean value,  $\bar{y}_i$ , during the interval,  $\Delta x_i$  
$$\bar{y}_i = \frac{y_{cum_i}}{\Delta x_i} = \frac{1}{c} \cdot \frac{\Delta y_i}{\Delta x_i}$$
- $$\bar{y}_i = y_i \cdot \frac{e^{c\Delta x_i} - 1}{c\Delta x_i} \quad \text{(replacing } \Delta y_i \text{ using Propriety 3)}$$
- $$\bar{y}_i = \frac{y_{i+1} - y_i}{\ln y_{i+1} - \ln y_i}$$
- $$\bar{y}_i \approx y_{central_i}$$

### ***Other useful expressions***

7. Expressions of variation,  $\Delta y_i$  
$$\Delta y_i = c \cdot y_{cum_i}$$
- $$\Delta y_i = c \cdot \bar{y}_i \Delta x_i$$
8. Expression of amr (y) 
$$amr(y) = \frac{\Delta y_i}{\Delta x_i} = c \cdot \bar{y}_i$$
9. Expression of rmr (y) in relation to  $\bar{y}_i$  
$$rmr(y)_{in\ relation\ to\ \bar{y}_i} = c = tir(y)$$
10. Expression of rmr (y) 
$$rmr(y) = amr(\ln y) = \frac{\Delta \ln y_i}{\Delta x_i} = c$$

11.  $y$  decreases
- $c < 0$
- $amr(y) < 0$
- $rmr(y) < 0$  and vice-versa
- If  $\Delta y_i < 0$  alors
- $$\left. \begin{array}{l} y_{cum_i} > 0 \\ \bar{y}_i > 0 \end{array} \right\}$$

12. In the exponential model, the geometric mean of  $y_i$  and  $y_{i+1}$  is equal to the central value,  $y_{\text{central}_i}$  (Prop. 4) and approximately equal to the mean value,  $\bar{y}_i$  (Prop. 6), been the approximation better when  $\Delta x_i$  is smaller.

### ***Demonstrations***

Cumulative value  
Property 5

$$y_{\text{cum}_i} = \int_{x_i}^{x_{i+1}} y \cdot dx = \int_{x_i}^{x_{i+1}} y_i \cdot e^{c(x-x_i)} \cdot dx = \left( \frac{y}{c} \right) \Big|_{x_i}^{x_{i+1}} = \frac{1}{c} \cdot \Delta y_i$$

Relation between  $\bar{y}_i$   
and  $y_{\text{central}_i}$

Property 6 –  
4<sup>th</sup> expression

From the approximation  $\frac{e^h - 1}{h} \approx e^{h/2}$  with  $h = c \cdot \Delta x_i$

and from property 6-2<sup>nd</sup> expression, will be:

$$\bar{y}_i \approx y_i \cdot e^{c \Delta x_i / 2}$$

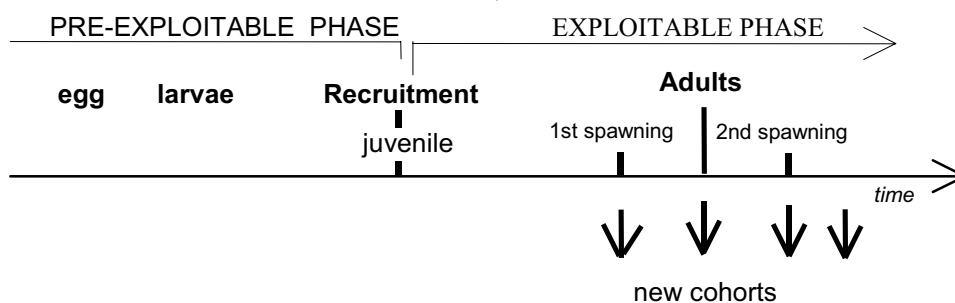
Finally, by property 4-1<sup>st</sup> expression, one can conclude that:

$$\bar{y}_i \approx y_{\text{central}_i}$$

# CHAPTER 3 – COHORT

## 3.1 COHORT – INTRODUCTION

A cohort or annual class or a generation, is a group of individuals born in the same spawning season. The following scheme illustrates the different phases of the life cycle of a cohort:



**Figure 3.1** Cycle of life of a cohort

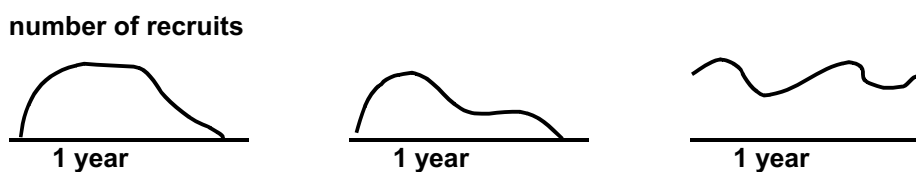
Let us start, for example, with the egg phase. The phases that follow will be larvae, juvenile and adult.

The number of individuals that arrive in the fishing area for the first time is called recruitment to the exploitable phase. These individuals grow, spawn (once or several times) and die.

After the first spawning the individuals of the cohort are called adults and in general, they will spawn again every year, generating new cohorts.

The phases of life of each cohort which precede the recruitment to the fishing area (egg, larvae, pre-recruits), are important phases of its life cycle but, during this time they are not usually subjected to exploitation. The variations in their abundances are mainly due to predation and environmental factors (winds, currents, temperature, salinity,...). *In these non exploitable phases mortality is usually very high, particularly at the end of the larvae phase (Cushing, 1996). This results in a small percentage of survivors until the recruitment. Notice that this mortality is not directly caused by fishing.*

The recruitment of a cohort during the exploitable phase, may occur during several months in the following schematic ways :



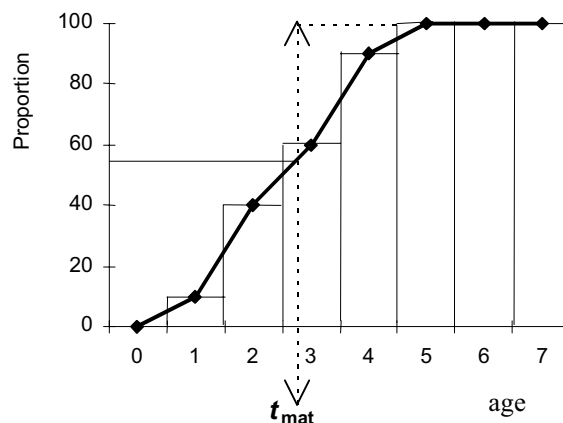
**Figure 3.2** Types of annual recruitment to the exploitable phase

With some exceptions, the forms of recruitment can be simplified by considering that all the individuals are recruited at a certain instant,  $t_r$  called age of recruitment to the exploitable phase. It was established that recruitments will occur on 1 January (beginning of the year in many countries). These two considerations do not usually change the results of the analyses, but simplify them and agree with the periods of time to which commercial statistics are referred.

It should be mentioned that not all the individuals of the cohort spawn for the first time at the same age. The proportion of individuals which spawn increases with age, from 0 to 100 percent. After the age at which 100 percent of the individuals spawned for the 1<sup>st</sup> time, all the individuals will be adult. The histogram or curve that represents these proportions is called maturity ogive.

In certain cases, the maturity ogive can also be simplified supposing that the 1<sup>st</sup> spawning occurs at the age  $t_{mat}$  designated as age of 1<sup>st</sup> maturity. This simplification means that the individuals with an age inferior to  $t_{mat}$  are considered juveniles and those with the same age or older, are considered adults.

Figure 3.3 represents a maturity ogive with the shape of a histogram or curve :



**Figure 3.3 Maturity ogive**

### **3.2 EVOLUTION OF THE NUMBER OF A COHORT, IN AN INTERVAL OF TIME**

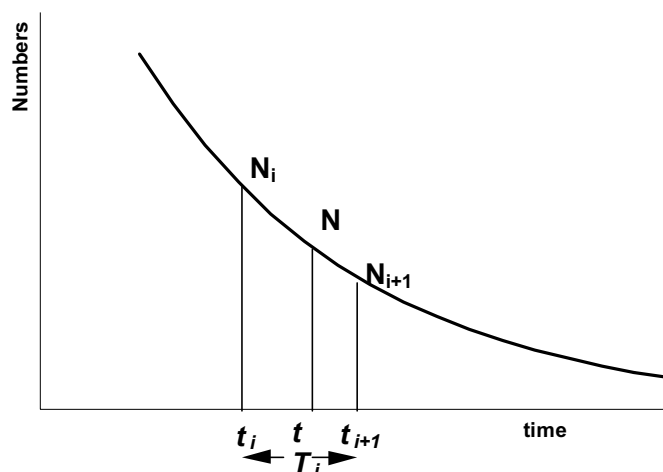
Consider the interval  $(t_i, t_{i+1})$  with the size  $T_i = t_{i+1} - t_i$  of the evolution of a cohort with time and  $N_t$  the number of survivors of the cohort at the instant  $t$  in the interval  $T_i$  (see Figure 3.4).

The available information suggests that the mean rates of percentual variation of  $N_t$  can be considered approximately constant, that is,  $rmr(N_t) \approx \text{constant}$ .

### **Basic assumption**

The relative instantaneous rate of variation of  $N_t$ , in the interval  $T_i$  is :

$$\text{rir}(N_t) = \text{constant negative} = -Z_i$$



$N_i$  = number of survivors at the beginning of the interval  $(t_i, t_i + 1)$

$N_{i+1}$  = number of survivors at the end of the interval  $(t_i, t_i + 1)$

**Figure 3.4 Evolution of N in the interval  $T_i$**

The model of the evolution of  $N_t$ , in the interval  $T_i$ , is an exponential model (because  $\text{rir}(N_t)$  is constant). This model has the following properties:

### **Properties**

1. General expression. From the basic assumption

$$\text{rir}(N_t) = -Z_i$$

with the initial condition that, for  $t = t_i$  it will be  $N_t = N_i$  then:

$$N_t = N_i \cdot e^{-Z_i(t-t_i)}$$

2. Number of survivors,  $N_{i+1}$ , at the end of interval  $T_i$

$$N_{i+1} = N_i \cdot e^{-Z_i T_i}$$

3. Number of deaths,  $D_i$ , during the interval  $T_i$

$$D_i = N_i - N_{i+1}$$

$$D_i = N_i(1 - e^{-Z_i T_i})$$

(notice that  $D_i$  is positive but the variation  $\Delta N_i = N_{i+1} - N_i$  is negative)

4. Cumulative number of survivors,  $N_{\text{cumi}}$ , during the interval  $T_i$

$$N_{\text{cumi}} = \frac{D_i}{Z_i}$$

$$N_{\text{cumi}} = N_i \cdot \frac{1 - e^{-Z_i \cdot T_i}}{Z_i}$$

5. Approximate central value,  $N_{\text{centrali}}$ , in the interval  $T_i$

$$N_{\text{centrali}} \approx N_i \cdot e^{-Z_i \cdot T_i / 2}$$

6. Mean number,  $\bar{N}_i$ , of survivors during the interval  $T_i$

$$\bar{N}_i = \frac{N_{\text{cumi}}}{T_i}$$

$$\bar{N}_i = N_i \cdot \frac{1 - e^{-Z_i \cdot T_i}}{Z_i \cdot T_i}$$

$$\bar{N}_i = \frac{D_i}{Z_i \cdot T_i}$$

$$\bar{N}_i = \frac{N_i - N_{i+1}}{\ln N_i - \ln N_{i+1}} \quad (\text{Ricker})$$

$$\bar{N}_i \approx N_i \cdot e^{-Z_i \cdot T_i / 2}$$

$$\bar{N}_i \approx N_{\text{centrali}} \quad \text{when } \frac{Z_i \cdot T_i}{2} \text{ is small } (Z_i \cdot T_i < 1)$$

### ***Comments***

1. The basic assumption is sometimes presented in terms of absolute instantaneous rates, that is:

$$\text{air}(N_t) = -Z_i \cdot N_t \quad \{ \text{air}(N_t) \text{ proportional to } N_t \} \text{ or}$$

$$\text{air}(\ln N_t) = -Z_i$$

$Z_i$  = mortality total coefficient, assumed constant at the interval  $T_i$

Notice that:

$$+Z_i = \text{rir of total mortality of } N_t$$

$$-Z_i = \text{rir of variation of } N_t$$

2. Unit of  $Z_i$

From the definition, it can be deduced that  $Z_i$  is expressed in units of  $[\text{time}]^{-1}$ . By agreement, the unit  $\text{year}^{-1}$  has been adopted, even when the interval of time is smaller or bigger than a year.

The following expressions show, in a simplified way, the calculation of the unit of  $Z_i$ , with the rules and usual symbols [...] of dimension in the determination of physical units .

$$\frac{[1]}{[N_t]} \cdot \frac{[dN_t]}{[dt]} = [-Z_i]$$

$$\frac{1}{\text{number}} \cdot \frac{\text{number}}{\text{time}} = +[Z_i] \quad \text{then} \quad [Z_i] = \text{time}^{-1}$$

### 3. Annual survival rate, $S_i$

When  $T_i = 1$  year, it will be :

$$N_{\text{cum}} = \bar{N}_i = \frac{D_i}{Z_i} \quad \text{and also}$$

$$N_{i+1} = N_i \cdot e^{-Z_i}$$

$S_i$  = Annual survival rate in the year  $i$   
(or *percentage of the initial number of individuals that survived at the end of the year*).

$$S_i = \frac{N_{i+1}}{N_i}$$

$$S_i = e^{-Z_i}$$

$1 - S_i$  = Annual mortality rate in the year  $i$

The percentage of the initial number of individuals that die during the year is, by definition, the relative mean rate  $\text{rmr}(N_t)$  of mortality of  $N_t$ , over one year, in relation to the initial number,  $N_i$

$$1 - S_i = \frac{D_i}{N_i} = 1 - \frac{N_{i+1}}{N_i} = \frac{N_i - N_{i+1}}{N_i}$$

$$1 - S_i = 1 - e^{-Z_i}$$

4. Absolute mean rate  $\text{amr}(N_t) = -Z_i \cdot \bar{N}_i$

5. Relative mean rate  $\text{rmr}(N_t) = -Z_i$  in relation to  $\bar{N}_i$

6. Notice that  $S_i$  takes values between 0 and 1 , that is:

$$0 \leq S_i \leq 1 \quad \text{but} \quad Z_i \text{ can be } > 1$$

7. If the limits of the interval  $T_i$  were  $(t_i, \infty)$  then it would be:

$$T_i = \infty$$

$$N_{i+1} = 0$$

$$D_i = N_i$$

$$N_{\text{cum}_i} = \frac{N_i}{Z_i} \quad \text{and}$$

$$\bar{N}_i = 0$$

### 3.3 CATCH, IN NUMBER, OVER AN INTERVAL OF TIME

The causes of death of the individuals of the cohort due to fishing will be separated from all other causes of death. These other causes are grouped together as one cause designated as natural mortality. So, from the properties of the exponential model, the result will be

$$\text{rir}(N_t)_{\text{total}} = \text{rir}(N_t)_{\text{natural}} + \text{rir}(N_t)_{\text{fishing}}$$

Supposing that, in the interval  $T_i$ , the instantaneous rates of mortality due to natural causes and due to fishing are constant and equal to  $M_i$  and to  $F_i$ , respectively, then

$$Z_i = F_i + M_i$$

Multiplying both factors of the previous equation (equality) by  $N_{\text{cum}_i}$ , then:

$$Z_i \cdot N_{\text{cum}_i} = F_i \cdot N_{\text{cum}_i} + M_i \cdot N_{\text{cum}_i}$$

$Z_i \cdot N_{\text{cum}_i}$  is the number,  $D_i$ , of deaths due to total mortality,

$$D_i = Z_i \cdot N_{\text{cum}_i}$$

In the same way  $F_i \cdot N_{\text{cum}_i}$  will be the number of dead individuals due to fishing, that is, the Catch,  $C_i$ , in number, and then:

$$C_i = F_i \cdot N_{\text{cum}_i}$$

Notice too that,  $M_i \cdot N_{\text{cum}_i}$  will be the number of dead individuals due to “natural” causes.

The exploitation rate,  $E_i$ , during the interval  $T_i$  was defined by Beverton and Holt (1956) as:



$$E_i = \frac{\text{number capture}}{\text{number dead}} = \frac{C_i}{D_i}$$

and then, 
$$E_i = \frac{F_i \cdot N_{\text{cum}_i}}{Z_i \cdot N_{\text{cum}_i}} = \frac{F_i}{Z_i} \quad \text{or}$$

$$E_i = \frac{F_i}{F_i + M_i}$$

The capture in number,  $C_i$ , in the interval  $T_i$ , can be expressed in the following different ways:

$$C_i = F_i \cdot N_{\text{cum}_i}$$

$$C_i = F_i \cdot \bar{N}_i \cdot T_i$$

$$C_i = E_i \cdot D_i$$

$$C_i = \frac{F_i}{Z_i} \cdot D_i$$

$$C_i = \frac{F_i}{F_i + M_i} \cdot N_i [1 - e^{-(F_i + M_i)T_i}]$$

### ***Comments***

1. Ricker (1975) defines the exploitation rate,  $E_i^*$ , as the percentage of the initial number that is captured in the interval  $T_i$ , that is:  $E_i^* = C_i / N_i$ .
  - a) Ricker's definition may be more natural, but mathematically Beverton & Holt's definitions are more useful.
  - b) It is easy to verify that  $E_i^* = E_i \cdot (1 - e^{-Z_i \cdot T_i})$
2. The exploitation rate,  $E_i$ , does not have any unit, it is an abstract number.
3. The possible values of  $E_i$  are between 0 e 1, being 0 when there is no exploitation and 1 when the capture  $C_i$  is equal to the number of total deaths  $D_i$ , that is, when  $M_i = 0$ .

### **3.4 INDIVIDUAL GROWTH**

In order to study the evolution of the biomass of a cohort, one can use the model of the evolution of a cohort, in number, and combine it with a model of the evolution of the mean weight of an individual of the cohort. In effect, the biomass  $B_t$  is equal to  $N_t \cdot W_t$  where  $W_t$  is the individual mean weight at the instant  $t$ .

To define a model for the individual growth weight  $W_t$ , there are then two possibilities :

**ALTERNATIVE 1:**

- A) To define a model for the mean individual growth in length,  $L_t$
- B) To define the relation Weight-Length.
- C) To combine A) with B) and obtain a mode for the mean individual growth in weight,  $W_t$

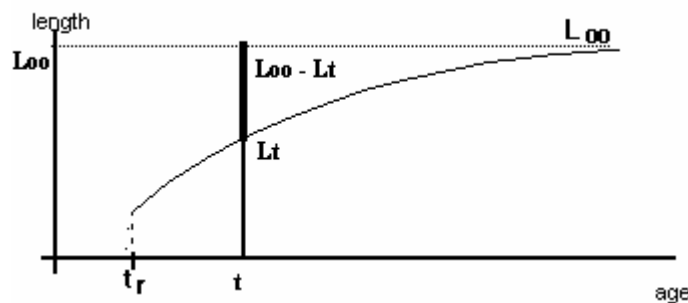
**ALTERNATIVE 2:**

- D) To define directly growth models for  $W_t$  and  $L_t$ .

**ALTERNATIVE 1**

**A) Model for the individual growth by length**

The models that are used in fisheries biology are valid for the exploitable phase of the resource. The most well known is the **von Bertalanffy Model** (1938) adapted by Beverton and Holt (1957). The existing observations suggest that there is an asymptotical length, that is, there is a limit to which the individual length tends.



**Figure 3.5 von Bertalanffy Model**

- $t$  - age
- $L_t$  - individual mean length at the age  $t$
- $L_{\infty}$  - asymptotical length
- $t_r$  - beginning of the exploitable phase

So,  $L_t$  presents an evolution where :

$\text{air}(L_t)$  is not constant (because growth is not linear)

$\text{rir}(L_t)$  is not constant (because growth is not exponential)

However, it can be observed that the variation of the quantity  $(L_{\infty} - L_t)$  (which we could call “what is left to grow”), presents a constant relative rate and can be described by an exponential model. So, we can adopt the:

### **Basic assumption**

$$\text{rir } (L_\infty - L_t) = -K = \text{negative constant during all the exploited life}$$

where K is the growth coefficient (attention: the growth coefficient K is not the velocity of growth but the relative velocity of what “is left to grow” !!).

The properties of this model can be obtained directly from the general properties of the exponential model. The initial condition :

$$t = t_a \quad \heartsuit \quad L_t = L_a$$

where  $t_a$  (and  $L_a$ ), that corresponds to an instant within the exploitable phase, will be adopted.

The properties of the model of individual growth by length by Beverton & Holt (1957) deduced from the exponential model of  $(L_\infty - L_t)$  are summarized as :

<b>Properties of the exponential model for <math>(L_\infty - L_t)</math></b>	<b>von Bertalanffy Model for <math>L_t</math></b>
<b>1. General expression</b>	
$(L_\infty - L_t) = (L_\infty - L_a) \cdot e^{-K \cdot (t - t_a)}$ <p>Parameters  <math>L_\infty =</math> asymptotic length  <math>K =</math> growth coefficient  <math>\left[ \begin{array}{l} t_a = \text{initial condition} \\ L_a \end{array} \right.</math></p>	$L_t = L_\infty - (L_\infty - L_a) \cdot e^{-K \cdot (t - t_a)}$ <p>Parameters  <math>L_\infty =</math> asymptotic length  <math>K =</math> growth coefficient  <math>\left[ \begin{array}{l} t_a = \text{initial condition} \\ L_a \end{array} \right.</math></p>
<p>For <math>t_a = t_0</math> and <math>L_a = 0</math> :</p> $(L_\infty - L_t) = L_\infty \cdot e^{-K \cdot (t - t_0)}$	<p>For <math>t_a = t_0</math> and <math>L_a = 0</math> :</p> $L_t = L_\infty - L_\infty \cdot e^{-K \cdot (t - t_0)}$ $L_t = L_\infty \cdot [1 - e^{-K \cdot (t - t_0)}]$
<b>2. Value at the end of the interval <math>T_i</math></b>	
$(L_\infty - L_{i+1}) = (L_\infty - L_i) \cdot e^{-K T_i}$	$L_{i+1} = L_\infty \cdot (1 - e^{-K T_i}) + L_i \cdot e^{-K T_i}$ <p>Ford-Walford expression (1933-1946)</p>

<p>3. <b>Variation during the interval <math>T_i</math></b></p> $\begin{aligned}\Delta(L_\infty - L_i) &= (L_\infty - L_{i+1}) - (L_\infty - L_i) \\ &= (L_\infty - L_i) \cdot e^{-k \cdot T_i} - (L_\infty - L_i) \\ &= (L_\infty - L_i) \cdot [e^{-k \cdot T_i} - 1]\end{aligned}$	<p>As <math>\Delta(L_\infty - L_i) = -\Delta L_i</math></p> <p>It will be::</p> $\Delta L_i = (L_\infty - L_i) \cdot (1 - e^{-k \cdot T_i})$
<p>4. <b>Cumulative value during the interval <math>T_i</math></b></p> $(L_\infty - L_i)_{\text{cum}_i} = \frac{\Delta(L_\infty - L_i)}{-K} = \frac{\Delta L_i}{K}$	$L_{\text{cum}_i} = L_\infty \cdot T_i - \frac{\Delta L_i}{K}$ <p>from:</p> $(L_\infty - L_i)_{\text{cum}_i} = L_\infty \cdot T_i - L_{\text{cum}_i} = \frac{\Delta L_i}{K}$
<p>5. <b>Mean value during the interval <math>T_i</math></b></p> $\overline{(L_\infty - L_i)} = \frac{(L_\infty - L_i)_{\text{cum}_i}}{T_i} = \frac{\Delta L_i}{K \cdot T_i}$	$\bar{L}_i = L_\infty - \frac{\Delta L_i}{K \cdot T_i}$ <p>Gulland e Holt expression (1959)</p> <p>from: <math>\overline{(L_\infty - L_i)} = L_\infty - \bar{L}_i</math></p>
<p>6. <b>Central value of the interval <math>T_i</math></b></p> $(L_\infty - L)_{\text{central}_i} = (L_\infty - L_a) \cdot e^{-K \cdot \left\{ \frac{t_{\text{central}_i} - t_a}{T_i} \right\}}$ <p>and</p> $(L_\infty - L)_{\text{central}_i} \approx \overline{(L_\infty - L_i)}$	$L_{\text{central}_i} = L_\infty - (L_\infty - L_a) \cdot e^{-K \cdot \left( \frac{t_{\text{central}_i} - t_a}{T_i} \right)}$ <p>and</p> $L_{\text{central}_i} \approx \bar{L}_i$

### ***B) Relation Weight-Length***

It is common to use the power function to relate the individual weight to the total (or any other) length. Then :

$$W_t = a \cdot L_t^b$$

where constant  $a$  is designated as condition factor and constant  $b$  as allometric constant. This relation can be justified accepting that the percentage of growth in weight is proportional to the percentage of growth in length, otherwise, the individuals would become disproportionate. Thus, the basic assumption is :

$$\text{rir}(W) = b \cdot \text{rir}(L)$$

where  $b$  is the constant of proportionality.

<b>C) Combination of A) and B) and comments:</b>	
1.	<p>From the combination of <math>W = a \cdot L^b</math> with <math>L_t = L_\infty \cdot (1 - e^{-K(t-t_0)})</math> we have</p> $W_t = W_\infty \cdot (1 - e^{-K(t-t_0)})^b \text{ with } W_\infty = a \cdot L_\infty^b$ <p>This relation of growth in weight is designated as the Richards equation (1959). When <math>b=3</math> the equation is the von Bertalanffy growth equation (1938).</p>
2.	<p>From, <math>W = a \cdot L^b</math> we have, by definition, <math>W_{\text{central}_i} = a \cdot L_{\text{central}_i}^b</math>, where <math>W_{\text{central}_i}</math> is the value corresponding to <math>L_{\text{central}_i}</math></p>
3.	<p>Let <math>\bar{w}_i^*</math> be the weight corresponding to <math>\bar{L}_i</math>, that is, <math>\bar{w}_i^* = a \cdot (\bar{L}_i)^b</math> As, <math>\bar{L}_i \approx L_{\text{central}_i}</math> then, <math>\bar{w}_i^* \approx a \cdot L_{\text{central}_i}^b = W_{\text{central}_i}</math></p> <p>In practice, <math>L_{\text{central}_i}</math> and <math>W_{\text{central}_i}</math> are preferred to the mean points</p>
4.	<p>The Richards and von Bertalanffy models are not the only models used for the evolution of <math>W_t</math>. Other models which have also been used in stock assessment are : Gompertz model (1825) and Ricker model (1969). (see Alternative 2 – property 3)</p>
5.	<p>Historically, the von Bertalanffy model was developed from the basic assumption</p> $\text{tia}(W_t) = \text{Cte}_1 \cdot W^{\frac{2}{3}} - \text{Cte}_2 \cdot W$ <p>Where <math>\text{Cte}_1</math> and <math>\text{Cte}_2</math> were designated by von Bertalanffy as the anabolism and the catabolism constants, respectively.</p> <p>Adopting the relation <math>W = a \cdot L^3</math> the basic assumption will become</p> $\text{air}(L_t) = \text{Cte}_1 - \text{Cte}_2 \cdot L$ <p>(where <math>\text{Cte}_1</math> e <math>\text{Cte}_2</math> are other constants).</p> <p>The solution of this differential equation gives the von Bertalanffy equation.</p>

## ALTERNATIVE 2

### ***D) Model directly for $W_t$ and $L_t$***

In alternative 1, a model was developed for growth in length, and then a model was built for growth in weight, using the relation between weight and length.

The basic assumption adopted by alternative 1 used the characteristic  $(L_\infty - L_t)$  instead of weight in order to be able to have a characteristic with a rate  $r$  constant, that is, an

exponential model. The relation  $W=a.L^b$  was adopted to obtain the model of growth in weight. Notice that it can be said that  $L$  was considered as a function of  $W$ , that is,  $L=(W/a)^{1/b}$ . It will then be possible to adopt, instead of that function of  $W$ , another function of the weight  $H(W_t)$ , in order to be able to formulate directly the basic assumption:

$$\text{air}[H(W_\infty) - H(W_t)] = -K = \text{constant}$$

with the initial condition

$$t = t_a \quad \heartsuit \quad W_t = W_a$$

### ***Properties***

The properties of this model (once it is an exponential model) can be obtained directly from the general properties of the exponential model. It is particularly interesting to derive the general expression  $W_t$  resulting from different choices of function  $H$ .

1. **General expression**  $[H(W_\infty) - H(W_t)] = [H(W_\infty) - H(W_a)] \cdot e^{-K(t-t_a)}$   
 or  $H(W_t) = H(W_\infty) - [H(W_\infty) - H(W_a)] \cdot e^{-K(t-t_a)}$

2a. **Richards equation in weight** Adopting the following function  $H(W_t) = W_t^{1/b}$   
 The result will be the general expression:  
 $W_t^{1/b} = W_\infty^{1/b} - (W_\infty^{1/b} - W_a^{1/b}) \cdot e^{-K(t-t_a)}$

that is, the Richards equation; and when  $b=3$ , this is the equation of von Bertalanffy, so:

2b. **von Bertalanffy equation in weight**, will be :  $W_t^{1/3} = W_\infty^{1/3} - (W_\infty^{1/3} - W_a^{1/3}) \cdot e^{-K(t-t_a)}$

3. **Gompertz equation in weight** Adopting the function  $H(W_t) = \ln W_t$   
 The result will be the general expression:  
 $\ln W_t = \ln W_\infty - (\ln W_\infty - \ln W_a) \cdot e^{-K(t-t_a)}$

4. The respective **equations in length** can be obtained by adopting other functions of  $H(W_t)$  :

4a. **von Bertalanffy equation in length** Adopting  $H(W_t) = L_t$  it will be:  
 $L_t = L_\infty - (L_\infty - L_a) \cdot e^{-K(t-t_a)}$

4b. **Gompertz equation de in length** Adopting  $H(W_t) = \ln L_t$  it will be:  
 $\ln L_t = \ln L_\infty - (\ln L_\infty - \ln L_a) \cdot e^{-K(t-t_a)}$

5. **Simplified equations**

The individual growth equations, both in length and in weight, are simplified when one selects  $H(W_a) = 0$  for  $t_a = t^*$

So, the simplified general expression will be reduced to :  $H(W_t) = H(W_\infty) \cdot (1 - e^{-K \cdot (t-t^*)})$

5a. **Simplified Richards equation, in weight**, will be :  $W_t = W_\infty \cdot [1 - e^{-K \cdot (t-t_0)}]^b$

where  $t^*$  was represented by  $t_0$  because  $H(W_a) = 0$  in Richards model means that  $W_a$  will also be zero.

5b. **Simplified Gompertz equation, in weight**, will be:  $\ln W_t = \ln W_\infty \cdot [1 - e^{-K \cdot (t-t^*)}]$   
In this case  $H(W_a) = 0$  corresponds to  $W_a = 1$

5c. **Simplified Richards equation in length**, will be:  $L_t = L_\infty \cdot [1 - e^{-K \cdot (t-t_0)}]$   
(with  $L_a = 0$  for  $t_a = t_0$ )

5d. **Simplified Gompertz equation, in length**, will be:  $\ln L_t = \ln L_\infty \cdot [1 - e^{-K \cdot (t-t^*)}]$   
(with  $L_a = 1$  for  $t_a = t^*$ )

**Comments**

1. Gompertz equation , in weight, is similar to Gompertz equation, in length, but, in their simplified forms,  $t^*$  represents different ages, because they will correspond, respectively, to  $W_a = 1$  and to  $L_a = 1$ .
2. Gompertz equation, in length, is similar to von Bertalanffy if  $L_t$  is substituted by  $\ln L_t$ . In practice, this fact allows the utilization of the same particular methods to estimate the parameters in both equations, using  $L$  in the von Bertalanffy expression and  $\ln L$  in the Gompertz expression. (See Section 7.4)
3. It is important to notice, once again that, in practice,  $L_{\text{centrali}}$  and  $W_{\text{centrali}}$  are used instead of the mean values,  $\bar{L}_i$  and  $\bar{W}_i$ .

4. Gompertz growth curve in length, has an inflection point,  $(t_{\text{infl}}, L_{\text{infl}})$ , with:

$$t_{\text{infl}} = t_a + (1/K) \cdot \ln(\ln(L_\infty/L_a)) \quad L_{\text{infl}} = L_\infty / e$$

5. Gompertz growth curve in weight has an inflection point,  $(t_{\text{infl}}, W_{\text{infl}})$  with:

$$t_{\text{infl}} = t_a + (1/K) \cdot \ln(\ln(W_\infty/W_a)) \quad W_{\text{infl}} = W_\infty / e$$

6. Richards growth curve in length does not have an inflection point but the growth curve in weight has an inflection point,  $(t_{\text{infl}}, W_{\text{infl}})$ .

$$t_{\text{infl}} = t_a - \frac{1}{K} \cdot \ln \left( \frac{1/b}{1 - \frac{W_a}{W_\infty}} \right)^{1/b} \quad W_{\text{infl}} = \left(1 - \frac{1}{b}\right)^b \cdot W_\infty$$

In the particular case of the von Bertalanffy equation it will be :

$$W_{\text{infl}} = (8/27) \cdot W_\infty \quad \text{and} \quad t_{\text{infl}} = t_0 + (1/k) \cdot \ln 3$$

7. Some authors refer Gompertz equation in other ways, for example, using the inflection point  $t_{\text{infl}}$  and the asymptotic weight  $W_\infty$ , instead of the parameters  $t_a$  and  $W_a$ .

It will then be  $w_t = w_\infty \cdot \exp(-e^{-k(t-t_{\text{inf}})})$  or  $L_t = L_\infty \cdot \exp(-e^{-k(t-t_{\text{inf}})})$

Sometimes the length expression is presented in its general form:  $L_t = a \cdot \exp(b \cdot e^{-c \cdot t})$

The parameters of the length model will then be:  $L_\infty = a$ ;  $k = -c$  et  $t_{\text{infl}} = (1/c) \cdot \ln(-b)$

8. The growth in length presents an inflection in fish farming, where the study of growth covers very young ages and it is common to use the Gompertz equation. In fisheries, the tradition is to use the von Bertalanffy equation.
9. A model that can sometimes be useful, is the Ricker model (1975). This model is valid for a certain interval of time  $T_i$  and not necessarily for all the exploitable life of the fishery resource. In fact, the model is based on the basic assumption that the individual growth is exponential in the interval  $T_i$ .

It will be, for example,  $L_t = L_i \cdot e^{K_i \cdot (t-t_i)}$  where  $K_i$  can be different from one interval to the next.

### 3.5 BIOMASS AND YIELD, DURING THE INTERVAL $T_i$

#### 1. Biomass

Theoretically, it could be said that the biomass at the instant  $t$  of the interval  $T_i$  is given by:

$$B_t = N_t \cdot W_t$$

Thus, the cumulative biomass during the interval  $T_i$  would be:

$$B_{\text{cum}_i} = \int_{t_i}^{t_{i+1}} B_t \cdot dt$$

and the mean biomass in the interval  $T_i$  would be:

$$\bar{B}_i = \frac{B_{\text{cum}_i}}{T_i}$$



In the same way, the mean weight of the cohort,  $\bar{W}_i$ , in the interval  $T_i$  would be:

$$\bar{W}_i = \frac{B_{cum_i}}{N_{cum_i}}$$

The biomass can be obtained by dividing both terms of the fraction by  $T_i$ , as

$$\bar{B}_i = \bar{N}_i \cdot \bar{W}_i$$

## 2. Yield

The yield,  $Y_i$ , during the interval  $T_i$  will be expressed as the product of the catch in numbers, times the individual mean weight :

$$Y_i = C_i \cdot \bar{W}_i$$

### *Comments*

In practice  $\bar{W}_i$  is considered approximately equal to  $W_{central_i}$  at the interval  $T_i$ .

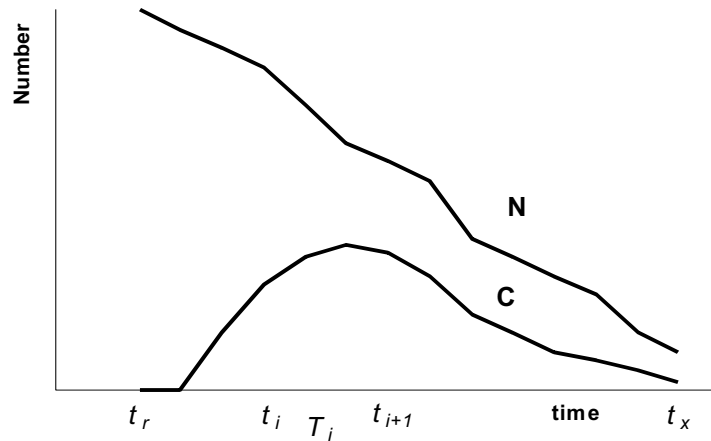
Other expressions of  $Y_i$  will also be :

$$\begin{aligned} Y_i &= F_i \cdot N_{cum_i} \cdot \bar{W}_i \\ &= F_i \cdot B_{cum_i} \\ &= F_i \cdot \bar{N}_i \cdot \bar{W}_i \cdot T_i \\ &= F_i \cdot \bar{B}_i \cdot T_i \end{aligned}$$

## 3.6 COHORT DURING THE EXPLOITABLE LIFE

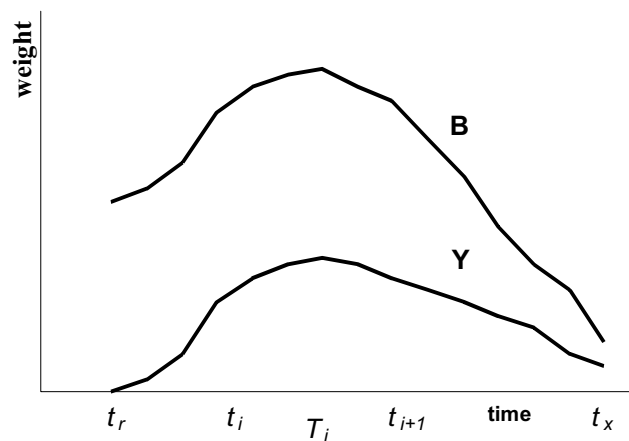
Consider the evolution of a cohort during the exploitable life, beginning at age  $t_0$  and intervals of time,  $T_i$ , covering all the exploitable phase (frequently the intervals are of 1 year...).

Figure 3.6 illustrates the evolution of the number of survivors of the cohort,  $\underline{N}$ , and the evolution of the catches in number,  $\underline{C}$ , which are obtained during the successive intervals of time  $T_i$ .



**Figure 3.6 Evolution of the number of survivors of the cohort, N, and the catches in number, C**

Figure 3.7 illustrates the evolution of the biomass of the cohort, B, and the evolution of the catches in weight, Y, which can be obtained during the successive intervals of time  $T_i$ .



**Figure 3.7 Evolution of the biomass of the cohort, B, and the catches in weight, Y**

**Values of the most important characteristics of the cohort,  
during all the exploitable phase**

Duration of the life of the cohort	$\lambda = \sum T_i$
Total number of deaths	$D = \sum D_i$ (= R (recruitment) when all the individuals die)
Cumulative number of survivors	$N_{cum} = \sum N_{cum_i}$
Mean number of survivors	$\bar{N} = N_{cum} / \lambda$

Cumulative biomass of survivors	$B_{cum} = \Sigma B_{cum_i}$
Mean biomass of survivors	$\bar{B} = B_{cum} / \lambda$
Catch in number	$C = \Sigma C_I$
Catch in weight	$Y = \Sigma Y_I$
Mean weight of the cohort	$\bar{W}_{coorte} = B_{cum} / N_{cum} = \bar{B} / \bar{N}$
Mean weight of the catch	$\bar{W}_{capt} = Y / C$
Critical age	$t_{critical}$ = age where B is maximum, when the cohort is not exploited.

### ***Comments***

1. At first sight it may seem that the values of the characteristics of a cohort during all the exploitable phase are of little interest, because, very rarely is fishing applied to an *isolated cohort*. At each moment, the survivors of several cohorts are *simultaneously* present and available for fishing.
2. Despite this fact and for reasons which will be mentioned later, it is important to analyse the characteristics of a cohort during all its exploitable life. Knowledge of the evolution of a cohort, in number and in biomass, and particularly the critical age, is important for the success of the activities in *fish farming*. As  $B_t = N_t \cdot W_t$ , the critical age,  $t_{critical}$ , will be the age  $t$  in the interval  $T_i$  where

$rir(W_t) = -rir(N_t) = M$  because, the critical age being the maximum biomass, the derivative of B will be equal to zero.

3. Notice that  $N_{cum}$  can be expressed in function of the recruitment as

$$N_{cum} = R \cdot \{...\}$$

where  $\{...\}$  represents a function of biological parameters and annual fishing mortality coefficients  $F_i$  during the life of the cohort.  $B_{cum}$  can also be expressed as:

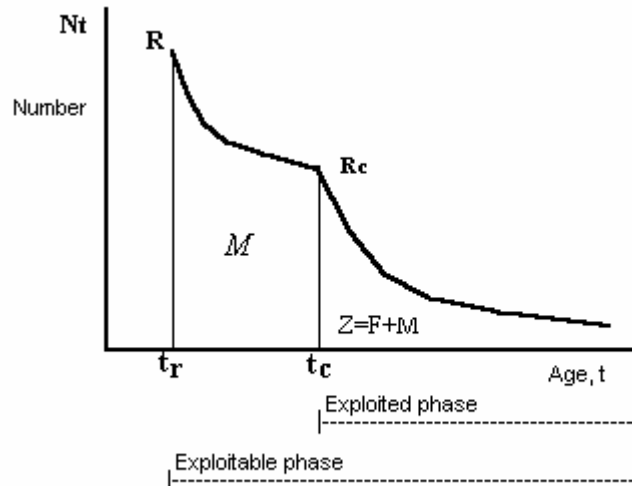
$$B_{cum} = R \cdot \{...\}$$

where the function  $\{...\}$  also includes growth parameters.

### 3.7 SIMPLIFICATION OF BEVERTON AND HOLT

Beverton and Holt (1957) deduced algebraic expressions for the characteristics of a cohort during the exploited life, adopting the simple assumptions:

1. The exploited phase of the cohort is initiated at age  $t_c$  and is extended to the infinite.
2. The natural mortality coefficient,  $M$ , is constant during all the *exploitable phase*.



**Figure 3.8 Life of a cohort during the exploitable phase**

3. The fishing mortality coefficient,  $F$ , is constant during all the exploited phase.
4. Growth follows the von Bertalanffy equation with for  $L_a = 0$  for  $t_a = t_0$

#### Basic equations referring to the exploited phase

1. **c (ratio between length at age  $t_c$  and the asymptotical length)**  $c = \frac{L_c}{L_\infty} = 1 - e^{-k \cdot (t_c - t_0)}$
2. **Recruitment**  $Y \quad R_c = R \cdot e^{-M(t_c - t_r)}$
3. **Cumulative number**  $N_{cum} = \frac{R_c}{Z}$
4. **Catch in number**  $C = E \cdot R_c$
5. **Cumulative biomass**

$$B_{cum} = R_c \cdot W_\infty \cdot \left\{ \frac{1}{M+F} - 3 \frac{(1-c)}{M+F+K} + 3 \frac{(1-c)^2}{M+F+2K} - \frac{(1-c)^3}{M+F+3K} \right\}$$

It can just be written

$$B_{cum} = R_c \cdot W_{\infty} \cdot [...]$$

6. **Mean weight in the catch**

$$\bar{W} = \frac{B_{cum}}{N_{cum}} = Z \cdot W_{\infty} \cdot [...]$$

7. **Catch in weight**

$$Y = C \cdot \bar{W} = F \cdot B_{cum} = F \cdot R_c \cdot W_{\infty} \cdot [...]$$

8. **Mean age in the catch**

$$\bar{t} = t_c + \frac{1}{Z}$$

9. **Mean length in the catch**

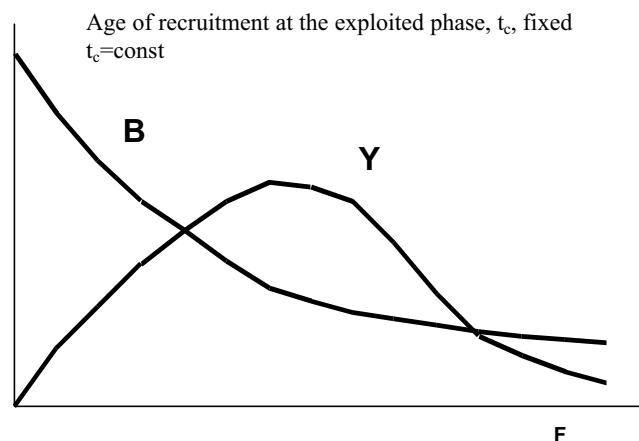
$$\bar{L} = L_{\infty} - (L_{\infty} - L_c) \cdot \frac{Z}{Z + K}$$

10. **Critical age**

$$t_{critique} = t_o - \frac{1}{K} \cdot \ln \left( \frac{M}{M + 3K} \right)$$

### Comments

1. The simplification of Beverton and Holt allows the calculation of any characteristic of the cohort during its life with algebraic expressions, avoiding the addition of the values of the characteristic in the successive intervals  $T_i$ . This was useful for calculations in the 60-70's when computers were not available. It is also useful when the only available data is natural mortality,  $M$ , and growth parameters.
2. At present, the simplified expressions are also useful to study the effects on the biomass, on yield and on the mean weight of the catch due to changes in the fishing mortality coefficient,  $F$ , and/or on the age of first capture,  $t_c$ . These analyses are usually illustrated with figures. For example, Figure 3.9 exemplifies the analyses of the biomasses and of the catches in weight, obtained from a cohort during the exploited life, subjected to different fishing mortality coefficients, assuming a fixed age  $t_c$ .



**Figure 3.9 Evolution of the biomass and of the catch in weight from a cohort subject to different fishing mortality coefficients and fixed  $t_c$**  (notice that the Figure illustrates only the analysis and does not take into consideration the scales of the axis).

3. Notice that the forms of the previous curves, Y and  $B_{cum}$  against F, do not depend on the value of the recruitment and so, they are usually designated as curves of biomass and yield per recruit, B/R and Y/R, respectively. The calculations are usually made with  $R=1000$ .
4. The mean weight, the mean age and the mean length of the catch do not depend on the value of the recruitment. The curves of the characteristics of a cohort during its life against the fishing level, F, or against the age of first catch,  $t_c$ , deserve a careful study, for reasons which will be stressed in the chapter concerning the long-term projections of the stock .
5.  $B_{cum}$  was calculated as:

$$B_{cum} = \int_{t_c}^{\infty} N_t \cdot W_t \cdot dt$$

where

$$N_t = R_c \cdot e^{-(M+F) \cdot (t-t_c)} \quad \text{and} \quad W_t = W_{\infty} \cdot \left[ 1 - e^{-K \cdot (t-t_0)} \right]^3$$

The calculations can also be made using other values of the constant b, from the relation W-L, different from the isomorphic coefficient,  $b=3$ , using the incomplete mathematical function Beta (Jones,1957).

6. The means  $\bar{L}$  and  $\bar{t}$  can be calculated from the cumulative expression

$$L_{cum} = \int_{t_c}^{\infty} N_t \cdot L_t \cdot dt \quad \text{and} \quad t_{cum} = \int_{t_c}^{\infty} N_t \cdot t \cdot dt \quad \text{as}$$

$$\bar{L} = L_{cum} / N_{cum} \quad \text{and} \quad \bar{t} = t_{cum} / N_{cum}$$

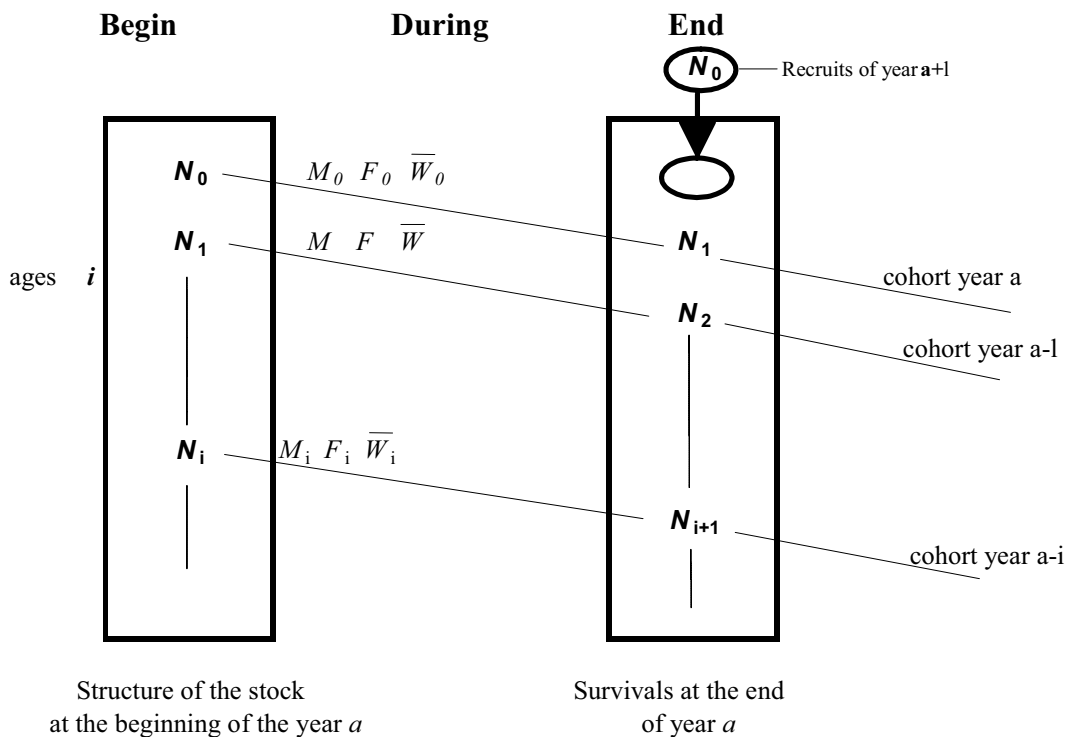
These means are designated as *weighted means*, being the *weighting factors*, the number of survivals,  $N_t$ , at each age t.

# CHAPTER 4 – STOCK

## 4.1 STOCK OVER A ONE YEAR PERIOD

### 4.1.1 EVOLUTION OF THE AGE STRUCTURE OF THE STOCK

Let us consider the evolution of a stock, with several cohorts, over the period of one year. Figure 4.1 shows the structure of the stock at the beginning of the year, the mean characteristics during the year and the age structure surviving at the end of the year.



**Figure 4.1** Structure of the stock at the beginning and end of the year

Let  $i = 0, 1, 2, 3, \dots$  be the ages

### 4.1.2 CHARACTERISTICS OF THE STOCK AT THE BEGINNING OF THE YEAR

Let :

- $N_i$  – number of individuals at age  $i$ , at the beginning of the year.
- $w_i$  - individual weight at age  $i$ , at the beginning of the year.
- $B_i$  - total biomass of the individuals at age  $i$ , at the beginning of the year.
- $N$  - total number of survivors of the stock at the beginning of the year.
- $B$  - total biomass of the stock at the beginning of the same year.

Then :

$$N = \sum_i N_i$$

$$B = \sum_i B_i = \sum_i (N_i \cdot W_i)$$

### 4.1.3 CHARACTERISTICS OF THE STOCK DURING THE YEAR

Let :

$M_i, F_i \in Z_i$  total mortality (natural and by fishing) coefficients, at age  $i$  during the year

$\bar{W}_i$  individual mean weight at age  $i$  during the year

$\bar{N}$  mean number of individuals during the year

$\bar{B}$  mean biomass during the year

$C$  catch, in number, during the year

$Y$  catch, in weight, during the year

$\bar{W}_{\text{catch}}$  mean weight of the individuals caught during the year

$\bar{W}_{\text{stock}}$  mean weight of the individuals of the stock during the year

Then:

$$N_{\text{cum}} = \sum_i N_{\text{cum}i}$$

$$\bar{N} = \frac{N_{\text{cum}}}{T} \quad (\text{where } T=1 \text{ year})$$

$$C = \sum_i C_i$$

$$B_{\text{cum}} = \sum_i B_{\text{cum}i}$$

$$\bar{B} = \frac{B_{\text{cum}}}{T} \quad (\text{where } T=1 \text{ year})$$

$$Y = \sum_i Y_i$$

$$\bar{W}_{\text{catch}} = \frac{Y}{C}$$

$$\bar{W}_{\text{stock}} = \frac{\bar{B}}{\bar{N}}$$

with  $N_{\text{cum}i}$ ,  $C_i$ ,  $B_{\text{cum}i}$  and  $Y_i$  calculated for all the ages  $i$  according to the expressions given previously in Chapter 3 - Cohort.



#### 4.1.4 CHARACTERISTICS OF THE STOCK AT THE END OF THE YEAR

The number of individuals at beginning of age  $i+1$  will be:

$$N_{i+1} = N_i \cdot e^{-Z_i}$$

Let :

R - recruitment of the cohort in the following year

Then the number and the biomass of the stock at the beginning of the following year will be :

$$N = R + \sum_i N_{i+1} \quad \text{and} \quad B = R \cdot W_1 + \sum_i B_{i+1}$$

where the product  $R \cdot W_1$  is the biomass of the recruitment of the following year.

#### *Comments*

1. The end of the year coincides with the beginning of the following year. So, the number of survivors of age  $i$  at the end of the year will be  $N_{i+1}$ , with age  $i+1$ .
2. The sum of the total number of survivors of the stock at the end of the year is not equal to the number of individuals of the stock at the beginning of the following year, because one has to count the recruits entering that year.
3. The total number of deaths, D, during the year, would be  $D = \sum_i D_i$
4. As the interval of time is 1 year, the cumulative values will be equal to the mean values, that is :

$$N_{\text{cum}_i} = \bar{N}_i$$

$$B_{\text{cum}_i} = \bar{B}_i$$

$$\bar{N} = \sum_i \bar{N}_i$$

$$\bar{B} = \sum_i \bar{B}_i$$

5. The utilization of the same symbols N, B, D, etc., for the stock and for the cohort should not create any confusions.

## 4.2 FISHING PATTERN OVER A ONE YEAR PERIOD

### 4.2.1 FISHING LEVEL AND EXPLOITATION PATTERN

The direct action of fishing a stock can be represented by the coefficients of mortality by fishing  $F_i$ . These coefficients are associated with the quantity of effort, with the disponibility of the individuals of different sizes or ages,  $i$ , and with the fishing gears used by the vessels during the year.

It is usual to separate the coefficients of mortality by fishing into two components:

One is designated as the level of intensity of mortality by fishing,  $\bar{F}$ , during the year, called fishing level,  $F$ . The level is associated with the quantity of fishing effort, (the number of vessels fishing), the number of days, hauls, fishing hours during the year, and with the efficiency or the fishing power of the vessels or gears. Another component, designated as exploitation pattern,  $s_i$ , is associated with the selective properties of the fishing gears relative to the sizes or ages of the individuals available to be captured, during that year.

The combined set of the fishing level (a unique value for all ages) and the exploitation pattern (different values according to the size or the age), is designated as fishing pattern or fishing regime. The designation of fishing pattern may cause some confusion with the exploitation pattern, and the designation of fishing regime may be confused with what the economists and managers call fishing regime.

The fishing pattern,  $F_i$ , of one age  $i$  during one year, is equal to the product of the fishing level of that year,  $\bar{F}$ , times the exploitation pattern of each age,  $s_i$ . That is:

$$F_i = \bar{F}.s_i$$

To analyse the effects of the coefficient of mortality by fishing,  $\bar{F}$ , on the characteristics of the resource and on the catches, the exploitation pattern is generally taken as constant from one year to the next. Sometimes one analyses the effects of the changes of the exploitation pattern maintaining a fixed fishing level, but one can also analyse the combined effects of the two components.

The fishing level,  $\bar{F}$ , is often represented by  $F$ .

### 4.3 SHORT-TERM PROJECTIONS OF THE STOCK

Knowing the structure of the stock at the beginning of one year, it is possible to estimate the characteristics of the stock during that year and project the structure of the stock for the beginning of the next year (with an exception to the recruitment of that year), for different values of the fishing level,  $\bar{F}$ , (and for the exploitation pattern,  $s_i$ ).

It is, of course, necessary to know the biological parameters of growth, maturity and the natural mortality coefficients, in each age during the year.

Adopting one value for the recruitment of the following year, the projection could be repeated for one more year and so on. The inconvenience in projecting the stock for several years will be that those projections will depend more and more on the adopted annual recruitment values. That is why, in practice, the stock and the catches are projected for one, or at the most, two years. In Section 4.5, the estimation of recruitments will be discussed.

Notice that to project a stock for the following year, it is necessary to have data from the previous year. So, the stock is firstly projected for the current year and then the catch and the biomass are projected for the following year.

Let us suppose, for example, that in 1997 one wants to project the characteristics of the stock for 1998. As the available data will be, in the best hypothesis, referring to 1996, one will have to project the stock of 1996 for the beginning of 1997 and together with the recruitment of 1997, project the stock until the end of 1997 and only after that, can one project to 1998, estimating previously the recruitment of 1998.

### 4.4 LONG-TERM PROJECTIONS OF THE STOCK

Let us consider the vector  $N$  with the components  $N_1, N_2, \dots, N_i, \dots$  representing the structure of the stock at the beginning of a year. Notice that  $N_1, N_2, \dots, N_i, \dots$  are the survivors of the different cohorts of the stock, at the beginning of the year.

Let  $M_i$  and  $F_i$  be the natural and fishing mortality coefficients of age  $i$ , assumed to be constant in the future years.

Figure 4.2 illustrates, with a theoretical stock, the projections of the numbers of survivors of the different cohorts at the beginning of 1980, for the years to come, from 1981-1986 (the values are expressed in million of individuals).

Notice that the recruitments are missing during the years 1981 to 1986, as they have not yet occurred. So, it is clear that the respective survivors are also missing during those years.

Let us suppose now that the recruitments of the same period of years were equal to those of 1980, that is, 440 million of individuals. Figure 4.3 shows the projections in future years. It can be seen that the values of the age structure of the stock in 1986 are equal to the annual evolution of the cohort in 1980.

Age	Year						
	1980	1981	1982	1983	1984	1985	1986
0	440						
1	995	326					
2	367	680	223				
3	68	229	425	139			
4	245	41	139	258	85		
5	345	149	25	84	156	51	
6	76	209	90	15	51	95	31

**Figure 4.2** Structure of the stock at the beginning of 1980 and projections of the number of survivors (million of individuals) of the different cohorts at the beginning of the years 1981-1986

Age	Year							
	1980	1981	1982	1983	1984	1985	1986	1987
0	440	440	440	440	440	440	440	440
1	995	326	326	326	326	326	326	326
2	367	680	223	223	223	223	223	223
3	68	229	425	139	139	139	139	139
4	245	41	139	258	85	85	85	85
5	345	149	25	84	156	51	51	51
6	76	209	90	15	51	95	31	31

**Figure 4.3** Long-term projections of the cohort of 1980 and structure of the stock assuming a constant annual recruitment of 440 million of individuals

One practical conclusion seems to be that to obtain the structure of the stock in 1986 it would be enough to follow the evolution of the cohort of 1980 and then, it would not be necessary to have the complete structure of the stock at the beginning of the year. It would be enough to adopt one value for the recruitment (  $R$  ) of a cohort (and, of course, assume that the biological parameters would be stable in the following years). An advice would be to make the calculations adopting a *Recruitment of 1000 individuals* (with computer software a recruitment equal to 1 is adopted).

Except for the mean values – age, length, weight in the catch, etc, – which are independent of the adopted value of  $R$ , the characteristics of the stock in the long-term, under the previous conditions, are proportional to the recruitment. So, these projections are also designated as *projections by recruit*, by *LT projections* or even as *equilibrium projections*.

The long-term projections do not only concern the survivors at the beginning of the year, but also the values of the mean abundances,  $\bar{N}_i$  during the year, and the catches in number,  $C_i$ .

One can also project the total biomasses,  $B$  or  $\bar{B}$ , the spawning biomasses,  $SB$  or  $\bar{SB}$ , as well as the catches in weight,  $Y$ , knowing the initial individual weights,  $W_i$ , the mean weights,  $\bar{W}$ , and other parameters like those of maturity.

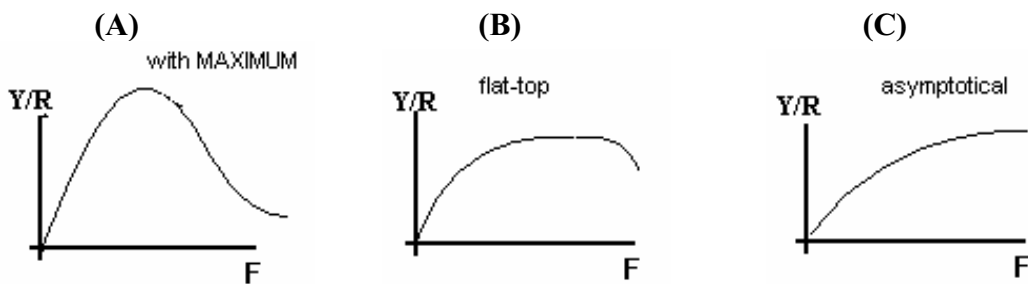
It is important to verify that the cumulative values,  $N_{cum_i}$  e  $B_{cum_i}$ , of a cohort during 1 year are equal to the mean values,  $\bar{N}_i$  e  $\bar{B}_i$ , during that year. The long-term projections can be calculated as:

$$\begin{aligned} \bar{N}_{stock} &= \bar{N}_{cum_i} = \bar{N}_i \\ \bar{B}_{stock} &= \bar{B}_{cum_i} = \bar{B}_i \quad ( \bar{SB}_{stock} = \bar{SB}_i ) \\ \bar{W}_{stock} &= \bar{B}_{stock} / \bar{N}_{stock} \\ C &= C_i \\ Y &= Y_i \\ \bar{W}_{catch} &= Y / C \end{aligned}$$

Several long-term projections can be made with different values of  $F_i$ , that is, with several fishing levels,  $\bar{F}$ , and/or with several exploitation patterns,  $s_i$ .

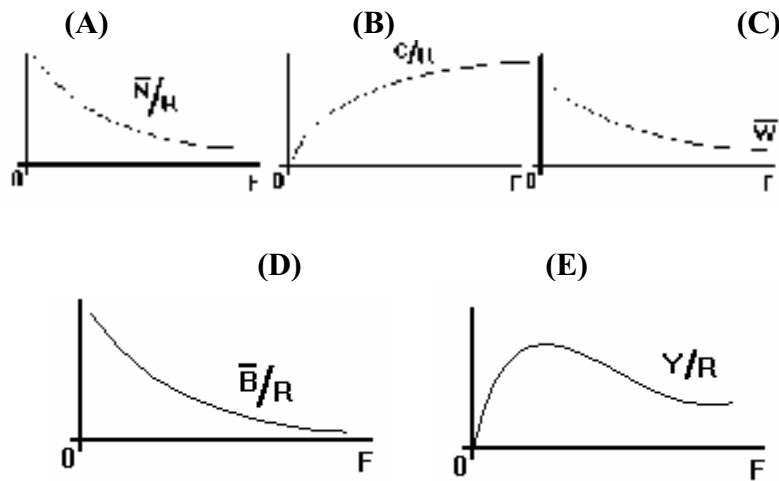
As mentioned before, the analyses of the effects of the fishing pattern on the catches and stocks can be done with the two components (fishing level and exploitable pattern), separated or combined.

Figure 4.4 (A-C) illustrates several types of yield per recruit curves, maintaining a fixed exploitation pattern. The curves are different for other exploitation patterns.



**Figure 4.4** Examples of types of curves of the Yield per Recruit (Y/R) against  $F$ , given the *exploitation pattern*: (A – with a maximum, B - flat-top, C – asymptotic)

Figure 4.5 (A-E) illustrates the relations between the most important characteristics of the stock and fishing level, maintaining a stable exploitation pattern:



**Figure 4.5** Relations between the long-term characteristics of the stock against the fishing level  $F$ , (A – mean number/ $R$ , B – catch in number/ $R$ , C - mean weight in the catch, D – mean biomass / $R$ , E – yield/ $R$ )

A more detailed analyses of these curves will be presented later on, in Chapter 5.

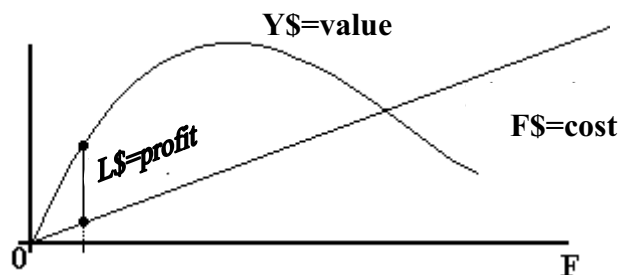
**Conclusion**

<p>The long-term structure of the stock by ages during 1 year</p>	<p>=</p>	<p>Evolution of the cohort during its life</p>
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**Comments**

1. One projects a cohort during its life in order to obtain the long-term projection of the stock for one year, assuming the annual recruitments to be constant.
2. It is necessary to know  $M_i$  for all ages of the cohort, as well as  $W_i$ ,  $\bar{W}$ ,  $s_i$  and the fishing level,  $\bar{F}$ , that is assumed to be constant in the years that follow.
3. Any recruitment size can be used. Adopt  $R = 1000$  (or  $R = 1$ ) with worksheets on your computer.
4. The five most important characteristics of the stock are  $\bar{N}_{stock}$ ,  $\bar{B}_{stock}$ ,  $C$ ,  $Y$  and  $\bar{W}_{catch}$  (see previous pages in this chapter for the respective expressions of calculation)
5. A characteristic of the stock that is also important is the spawning biomass,  $SB$ . To calculate  $SB$ , it is necessary to know the maturity ogive (or histogram).
6. Long-term projections are also designated as equilibrium situations.
7. Long-term projections are useful to define the long-term management objectives.

8. Annual  $\bar{W}_{\text{catch}}$ , are independent of the recruitment size (such as  $\bar{L}_{\text{catch}}$  and  $\bar{t}_{\text{catch}}$ ).
9. Economists transform the *total yield*,  $Y$ , into *value*  $Y\$,$  *the mean weight of the catch*,  $\bar{W}_{\text{catch}}$  into price of the catches,  $\bar{W}\$,_{\text{catch}}$ , *the catch per vessel (or the cpue)* into value of the production by vessel,  $U\$,$  and the *fishing level*,  $\bar{F}$  into costs of exploitation,  $F\$,$ . The *difference between the value of the catch and the cost of exploitation*,  $Y\$-F\$,$  is the profit,  $L\$,$ . Figure 4.6 illustrates an example of the LT relations between those characteristics used by the economists against the fishing level,  $\bar{F}$ .



**Figure 4.6** Long-term relations between the value of the catch, the cost of exploitation and the profit against the fishing level

#### 4.5 STOCK-RECRUITMENT (S-R) RELATION

The stock-recruitment relation, known by S-R relation, associates the size of the stock, for one year, with *the recruitment which results* from the stock spawning normally during that year. The recruitment can be the recruitment at the exploitable phase and the stock can be the spawning stock. This recruitment may occur one or more years after the spawning.

The problem with the relation between the parental population and the new generation is not a special case of the fisheries resources, it is common to all the self renewable populations.

It is important to determine, in each case, the stock and the recruitment to be used. In fact, that *stock* can be the total number of individuals (at the beginning of the year or the mean value during the year), the number of adult individuals of the stock, the number of adult females, etc. One can also adopt, not the abundances in number, but the corresponding biomasses. The decision will depend on the type of resource and on the available data. It is necessary to define if the recruitment is in weight or in number and if the recruitment is to the fishing area or to the exploited phase.

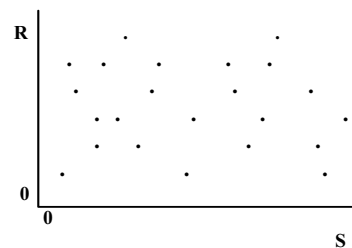
In this manual, stock (S) will be considered as the spawning biomass and the recruitment (R) will be expressed in number.

After the spawning, the individuals of the new generation will have to go through different phases of the biological cycle: eggs, larvae and juveniles, before they become recruits. These phases, which, in some species can take years, are not directly submitted to the fisheries exploitation. That is why fishing in those years does not directly affect the size of the new

recruitment. It is true that fishing acts on the size of the parental biomass, but that does not happen in the pre-recruit phases, and it is precisely for that interval of time that the relation S–R is applicable.

There is, in these pre-recruit phases, a great mortality due to climate and environmental factors (winds, currents, temperatures, etc.) as well as due to biological factors (available food, predation and others).

A great variety of factors (besides fishing) cause great fluctuations to the recruitment sizes, therefore, the relation S–R is a complex one. In conclusion, **the relation S–R, between the stock and the resulting recruitment, can be considered independent of fishing.**



**Figure 4.7 Example of the dispersion of the points in the relation S–R**

Figure 4.7 shows the type of dispersion of values in the plot of recruitment (R) against parent stock (S).

Despite the difficulties, some models have been proposed for the relation S–R. One of the reasons for this is that there must be a relation. One point of the relation (S = 0, R = 0) is even known already.

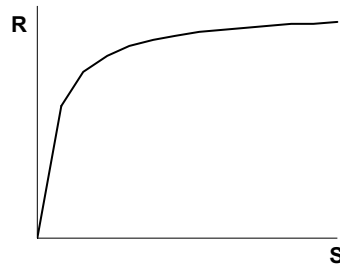
#### **4.5.1 BEVERTON AND HOLT MODEL (1957)**

Beverton and Holt, when analysing plaice fishery in British waters proposed one model, which can be re-written as:

$$R = \frac{\alpha \cdot S}{1 + \frac{S}{k}}$$

where  $\alpha$  and  $k$  are constants.





**Figure 4.8 Beverton and Holt Relation S–R**

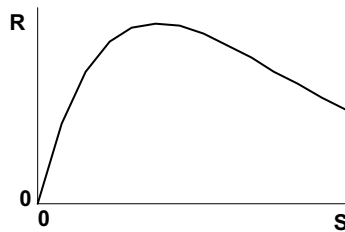
This model shows a curve where R is asymptotic when  $S \rightarrow \infty$ .

#### 4.5.2 RICKER MODEL (1954)

Ricker, studying cod fishery in Canadian waters proposed another model that can be written as:

$$R = \alpha.S.e^{-S/k}$$

where  $\alpha$  and  $k$  are constants.



**Figure 4.9 Ricker Relation S–R**

This model presents a maximum resulting recruitment at an intermediate value of the parental stock, S.

#### 4.5.3 OTHER MODELS

Other models of S–R have been proposed, like the **Deriso generalized model (1980)**, which can be re-written (Hilborn & Walters, 1992) as :

$$R = \alpha.S.(1 - c.\frac{S}{k})^{1/c}$$

(the model is the Beverton and Holt for  $c = -1$  and when  $c \rightarrow 0$  it is the Ricker model)

or **Shepherd's generalized model (1982)**, as:

$$R = \frac{\alpha \cdot S}{1 + \left(\frac{R \cdot S}{TM_k}\right)^c}$$

(the model is the Beverton and Holt when  $c = 1$  and when  $c = 2$  the curve is an approximation of the Ricker model)

Notice that in every model presented,  $S=0$  implies  $R=0$ , as expected and the slope of the tangent to the curve at that point,  $(0,0)$ , is equal to the parameter  $\alpha$ . Sometimes, the relations  $S-R$  are presented as  $R/S$  in function of  $S$ , so the Beverton and Holt equation would be :

$$\frac{R}{S} = \frac{\alpha}{1 + \frac{S}{k}}$$

showing that the inverse of  $R/S$  is linear with  $S$ .

With the Ricker model the relation between  $R/S$  against  $S$  would be:

$$\frac{R}{S} = \alpha \cdot e^{-S/k}$$

that is,  $R/S$  is exponential negative with  $S$  (or  $\ln(R/S)$  is linear with  $S$ ).

With the Deriso model it would be  $(R/S)^c$  linear with  $S$ .

Mathematically, these linear relations can be useful to estimate the parameters  $\alpha$  and  $k$ , but statistically they cause some problems, because the variable  $S$  appears in the response variable and in the auxiliary variable.

### **Comments**

1. Remember that relations  $S-R$  are independent of the fishing level.
2. Relations  $S-R$  may be introduced in the calculations of the stock projections. In that case, the projections will have to be made every year and they will require the structure of the stock at the beginning of the initial year.
3. It must not be forgotten that there is a great dispersion of the observed points  $(S, R)$  around the model.
4. To determine the limit of the Deriso equation when  $c \rightarrow 0$  it is enough to remember that  $\lim_{n \rightarrow \infty} (1+A/n)^n = e^A$  when  $n \rightarrow \infty$  and  $A$  is a constant.
5. The Beverton and Holt model presents an asymptote,  $R \rightarrow \alpha \cdot k$  when  $S \rightarrow \infty$ . When  $S=k$  it will be  $R = \alpha k/2$ .

The Deriso model presents the maximum:

$$S_{\max} = k / (1+c), R_{\max} = \alpha \cdot k / (1+c)^{1+(1/c)}$$

The Shepherd model presents the maximum:

$$S_{\max} = k.(c-1)^{-1/c}, R_{\max} = (\alpha /c).k.(c-1)^{1-(1/c)}$$

#### 4.6 RELATION BETWEEN R AND $\bar{B}$ (R-S RELATION)

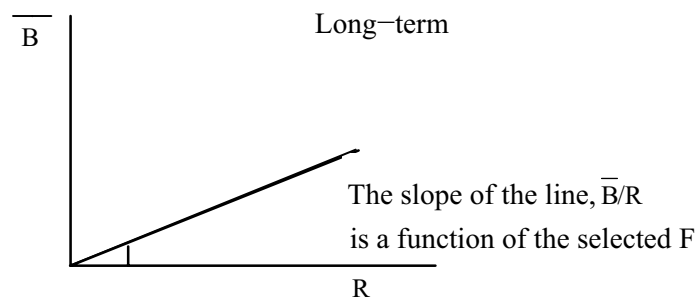
Up to now the discussion have been centred on the relation S-R, that is the relation between the biomass S and the resulting recruitment R. There is another relation (which has already been referred to in Section 4.4 Long-term projections of the stock, particularly in the conclusion about the structure of the stock and the evolution of a cohort during all its life) that could be called the relation R-S, that is the relation between the recruitment R and the **resulting cumulative biomass**,  $B_{\text{cum}}$  of a cohort during all its life for a given fishing pattern.

The cumulative biomass (or spawning biomass) of a cohort during its life,  $B_{\text{cum}}$ , is, as it has already been seen, equal to:

$$B_{\text{cum}} = R \cdot \{ \text{function of biological parameters and of the fishing pattern, } F \}$$

As mentioned before, the cumulative biomass,  $B_{\text{cum}}$ , of a cohort during its life is equal to the long-term *mean biomass*,  $\bar{B}$ , of the stock during one year, so, one can refer to the  $B_{\text{cum}}$  as  $\bar{B}$ .

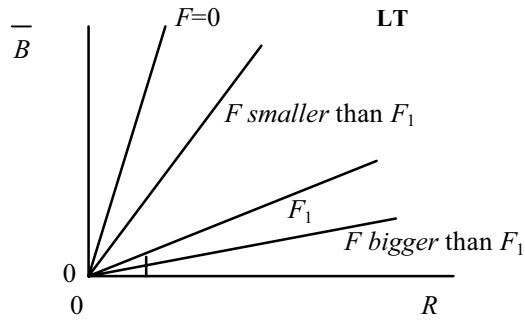
It can then be said that in the long-term, if the biological parameters are considered constant, then for a certain fishing pattern, the mean biomass of a stock during one year is proportional to the recruitment. Figure 4.10 illustrates the proportionality.



**Figure 4.10** Illustration of the proportionality in the long-term between the mean Biomass  $\bar{B}$  of a stock and the Recruitment (R), for a certain fishing level, F, assuming that the exploitation pattern and the biological parameters are constant

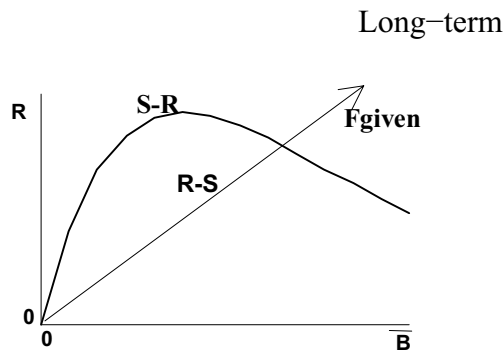
Notice that in Figure 4.5-D, the relation  $B/R$  against  $F$  was shown. The two curves 4.5-D and that of Figure 4.10 are different representations of the same situation. While in Figure 4.5-D, a value of  $F$  corresponded to a value of  $B/R$  given by the curve, in Figure 4.10 the value of  $F$  will correspond to a slope  $B/R$  of a given straight line.

For other values of  $F$ , the straight line of Figure 4.10 will have different inclinations. Figure 4.11 shows several lines according to the different values of  $F$ .



**Figure 4.11 Illustration of four lines corresponding to different fishing levels, F**

Figure 4.12 shows the overlapping of the Ricker curve S-R (Figure 4.9) and the line in Figure 4.10, the axes of this last figure having been switched. To avoid confusions, it is convenient to refer the slope of the line corresponding to a value F in relation to the axis R.

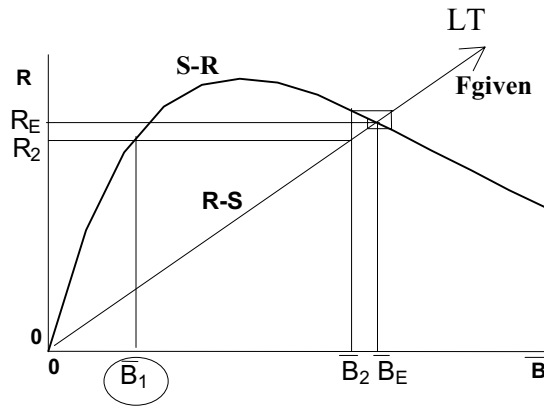


**Figure 4.12 Overlapping of the curve S-R with the line R-S for a given F**

One can start from a value of biomass, and, through the relation S-R, determine the future recruitment, R. That R will give a resulting biomass,  $\bar{B}$ , through the straight line. This process can be repeated until a situation of equilibrium is found.

It will be possible to illustrate the combined analysis through the two relations for a certain fishing level.

For example, let us select one value  $\bar{B}_1$ . The curve S-R allows the calculation of the resulting value  $R_2$ . Through the line (of a given F), the resulting value of  $\bar{B}_2$  will correspond to that value of  $R_2$ , and so on. Figure 4.13 shows that the process reaches the equilibrium point  $(R_E, \bar{B}_E)$ . This point would correspond to a value of the fishing level F, determined by the slope of the line in relation to the R axis.



**Figure 4.13** Illustration of the process that, starting from a biomass  $\bar{B}_1$ , theoretically leads to the equilibrium point  $(R_E, B_E)$

Notice that the intersection of the two relations does not always lead to an equilibrium point. The existence of an equilibrium point depends on the angle between the straight line and the curve at the intersection point.

# CHAPTER 5 – BIOLOGICAL REFERENCE POINTS AND REGULATION MEASURES

## 5.1 BIOLOGICAL REFERENCE POINTS FOR THE MANAGEMENT AND CONSERVATION OF FISHERIES RESOURCES

The long-term objectives for fisheries management should take into consideration scientific fishing research and population dynamics, as well as the climatic changes that may affect the stocks.

In order to define these long-term objectives we have to consider the values of the fishing level, which allow bigger catches in weight, whilst also ensuring the conservation of the stocks. The extreme values of the biomass or the fishing level, which might seriously affect the self renovation of the stocks, also have to be considered. These fishing level values, of catch and biomass are designated as biological reference points (BRP). In this manual some of the different types of BRP will be considered (Caddy, & Mahon, 1995; FAO, 1996 and ICES, 1998).

The Target Reference Points, TRP are BRP defined as the level of fishing mortality or of the biomass, which permit a long-term sustainable exploitation of the stocks, with the *best possible catch*. For this reason, these points are also designated as *Reference Points for Management*. They can be characterized as the *fishing level*  $F_{\text{target}}$  (or by the *Biomass*,  $B_{\text{target}}$ ).

The most well known  $F_{\text{target}}$  is  $F_{0.1}$  but other values, like  $F_{\text{max}}$ ,  $F_{\text{med}}$ , and  $F_{\text{MSY}}$  will also be studied.

For practical purposes of management, the TRP will be converted, directly or indirectly, into values of fishing effort, given as percentages of those verified in recent years.

The Limit Reference Points, LRP are maximum values of fishing mortality or minimum values of the biomass, which must not be exceeded. Otherwise, it is considered that it might endanger the capacity of self-renewal of the stock.

In the cases where fishing is already too intensive, the LRP can be important to correct the situation or to prevent it from getting worse.

The LRP are limit values, mainly concerned with the conservation of marine stocks and they are therefore also referred to as *reference points for conservation* (this designation does not imply that the  $F_{\text{target}}$  are not concerned with conservation).

Several LRP have been suggested, which will generally be referred to as  $F_{\text{lim}}$  or  $B_{\text{lim}}$ . In this manual the levels of biomass  $B_{\text{loss}}$  and MBAL will be referred to as  $B_{\text{lim}}$  and the fishing levels  $F_{\text{loss}}$  and  $F_{\text{crash}}$  as  $F_{\text{lim}}$ .

The Precautionary Principle, proposed by FAO in the Conduct Code for Responsible Fisheries (FAO, 1995), declares that the limitations, uncertainties or lack of data for the assessment or for the estimation of parameters, cannot be justification for not applying regulation measures, especially when there is information that the stocks are over-exploited.

From this point of view, it is important to make clear which basic assumptions are necessary in order to estimate the consequences on the catches and on the abundance of the stocks.

The uncertainties associated with the estimation of  $F_{lim}$ , and  $B_{lim}$ , therefore lead us to determine new reference points, called Precautionary Reference Points,  $F_{pa}$  or  $B_{pa}$ .

The assumptions and the consequences of adopting alternative hypotheses about the stock and fishing characteristics should always be presented to justify the estimated values of  $F_{pa}$  (or  $B_{pa}$ ).

The new limits ( $F_{pa}$  or  $B_{pa}$ ) due to the application of the Precautionary Principle, will be more restrictive than the LRP's. The practical consequences of these new limits are the regulation measures designed to control the fishing effort which are more severe than in those cases where there is appropriate data.

It can be said that this is the price to pay for not having the appropriate conditions to make available reliable data and information.

The Precautionary Approach, suggests that the results of fisheries research should be adopted by management with regard to the formulation of the regulation measures and that these measures should also take into consideration the *socio-economic and technical conditions of fishing* (FAO, 1996).

A final remark about all the Biological Reference Points mentioned above :

The evaluation of the biological reference points has to be updated, taking into consideration the possible changes in the biological parameters or any other necessary correction of the exploitation pattern. This fact is important because the new biological reference points will be different from the previous ones.

## 5.2 BIOLOGICAL TARGET REFERENCE POINTS ( $F_{\max}$ , $F_{0.1}$ , $F_{\text{med}}$ and $F_{\text{MSY}}$ )

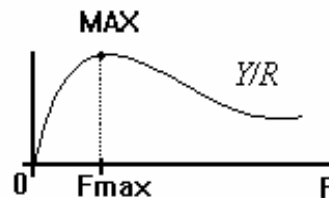
### 5.2.1 $F_{\max}$

#### *Definition*

1. Consider the long-term yield per recruit,  $Y/R$ , as a function of  $F$ , for a certain exploitation pattern.

$F_{\max}$  is the point of the curve,  $Y/R$  against  $F$ , where  $Y/R$  is maximum.

Figure 5.1 shows a curve of  $Y/R$  against  $F$ .



**Figure 5.1**  $Y/R$  as function of  $F$  for a certain  $t_c$  constant, showing  $F_{\max}$  and  $Y_{\max}$

In Chapter 4 it was mentioned that to estimate the long-term projections one could assume that the recruitment is constant and equal to 1 ( $R=1$ ). In this way, the mathematical expressions are sometimes written with  $Y$  instead of  $Y/R$ .

2. Mathematically, at point  $F_{\max}$ , the derivative of  $Y/R$  against  $F$  is equal to zero, that is,

$$\text{For } F = F_{\max} \text{ will be } \frac{dY}{dF} = 0 \quad \text{air}(Y) = 0 \text{ (Value of } Y \text{ is maximum)}$$

$$\text{For } F < F_{\max} \text{ will be } \frac{dY}{dF} > 0 \quad \text{air}(Y) > 0 \text{ (} Y \text{ is increasing with } F \text{)}$$

$$\text{For } F > F_{\max} \text{ will be } \frac{dY}{dF} < 0 \quad \text{air}(Y) < 0 \text{ (} Y \text{ is decreasing with } F \text{)}$$

Geometrically, the slope of the tangent to the curve is equal to zero for  $F = F_{\max}$ , positive for  $F < F_{\max}$  and negative for  $F > F_{\max}$ .



## Comments

1.  $\bar{B}_{\max}/R$  and  $Y_{\max}/R$  are the values at  $F_{\max}$ .

It is also convenient to analyse the situation of  $\bar{B}/R$  at the points  $F \neq F_{\max}$ .

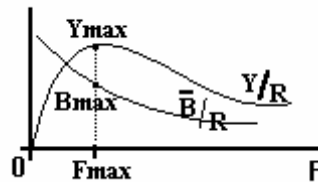
$F < F_{\max}$  corresponds to  $\bar{B} > \bar{B}_{\max}$

$F > F_{\max}$  corresponds to  $\bar{B} < \bar{B}_{\max}$

Point  $F_{\max}$  does not depend on the value of the recruitment.

2. For another relative pattern of exploitation there will be another  $F_{\max}$ .
3. All the points of the curve  $Y/R$  against  $F$ , are long-term points or equilibrium points.
4. When the level,  $F$ , is bigger than  $F_{\max}$  it is said that there is growth overfishing.

It is convenient to present the two curves  $Y/R$  and  $\bar{B}/R$  against  $F$ , in the same graph (usually with different scales).



**Figure 5.2** Long-term curves of  $Y/R$  and  $\bar{B}/R$  against  $F$ , given an exploitation pattern

$F_{\max}$  was adopted by the majority of the International Fisheries Commissions as a long-term objective of management (1950-1970).

Even today  $F_{\max}$  is used as a target-point having been proposed as a Limit Reference Point (LRP) in some cases.

The flat-top and asymptotical curves do not allow the determination of an  $F_{\max}$ .

The definition of  $F_{\max}$  does not consider the appropriate level of spawning biomass.

$F_{\max}$  only indicates the value of  $F$  which gives the maximum possible yield per recruit from a cohort during its life, for a given exploitation pattern.

The analyses of these long-term curves, mainly of  $\bar{B}$ ,  $Y$  and  $\bar{W}_{\text{catch}}$  against the *fishing level*, give information about the abundance of the resource (or catch per vessel), total yield of all the fleet and mean catch weight for different fishing levels.

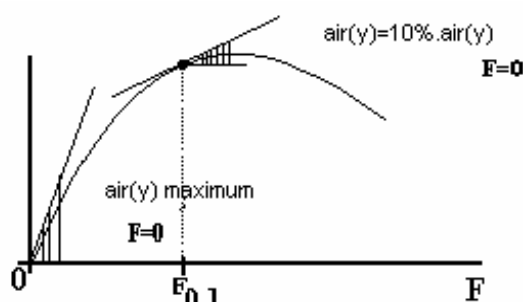
## 5.2.2 $F_{0.1}$

### 1. **Definition**

Consider the long-term yield per recruit,  $Y/R$ , as a function of the coefficient of fishing mortality,  $F$ . One value of  $\text{air}(Y/R)$ , corresponds to each fishing level,  $F$ . The  $\text{air}(Y/R)$  is maximum when  $F = 0$  and decreases, being zero when  $F = F_{\max}$ .

The point  $F_{0.1}$  is the value of  $F$  where  $\text{air}(Y/R)$  is equal to 10 percent of  $\text{air}(Y/R)$  maximum.

The figure 5.3 illustrates this situation.



**Figure 5.3**  $Y/R$  showing the reference target point  $F_{0.1}$

2. For  $F = 0$ , the biomass per recruit,  $\bar{B}/R$  will be  $\bar{B}_0/R$ , also designated as *Virgin Biomass* or *Non-exploited Biomass*. The  $\text{air}(y)$  at  $F=0$  is also equal to  $B_0$

In fact,  $Y = F \cdot \bar{B}$  implies  $\frac{dY}{dF} = \bar{B} + F \frac{d\bar{B}}{dF}$

Then, for  $F = 0$ ,  $\text{air}(Y) = \left. \frac{dY}{dF} \right|_{F=0} = \bar{B}_0$

So, from the definition given in point 1 one can also say that  $F_{0.1}$  is the value of  $F$  where  $\text{air}(Y) = 10$  percent of the virgin biomass.

### 3. **Calculation of $F_{0.1}$**

Let the function  $V = Y - 0.1 \cdot \bar{B}_0 \cdot F$

It can be proved that the function  $V$  is maximum when  $F = F_{0.1}$

In fact,  $V$  is maximum when  $\frac{dV}{dF} = 0$ , then:

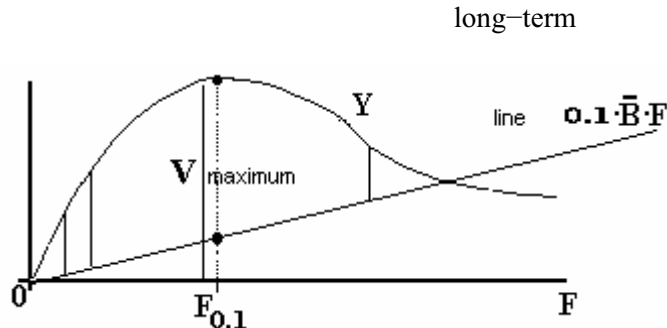
$$\frac{dV}{dF} = \frac{dY}{dF} - 0.1 \cdot \bar{B}_0 = 0 \quad \text{or} \quad \frac{dY}{dF} = 0.1 \cdot \bar{B}_0$$

Therefore, the value of  $F$  corresponding to the previous  $dY/dF$  is the value of  $F_{0.1}$ .

$F_{0.1}$  can then be calculated by maximizing the function  $V = Y - 0.1 \cdot \bar{B}_0 \cdot F$

$\bar{B}_0$  can be calculated, for example, from the long-term relation of  $\bar{B}$  against  $F$ , when  $F = 0$ .

Graphically it will be:

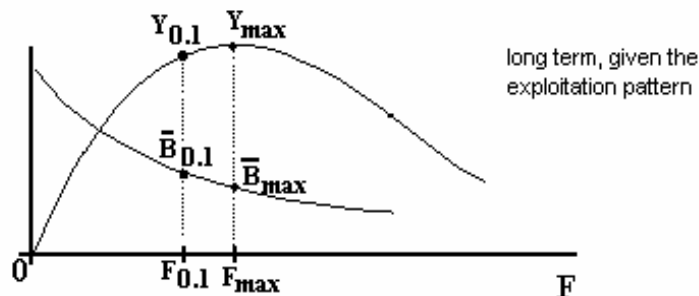


**Figure 5.4** Curve  $Y/R$  showing the maximum of the function  $V$

4. Why adopt  $\text{air}(Y/R)$  equal to 10 percent and not any other percentual value, for example, 20 percent?

Gulland and Boerema (1969) presented some arguments, including financial arguments. Some countries (like South Africa) adopt the value of 20 percent with a resulting reference point  $F_{0.2}$  that is more restrictive than  $F_{0.1}$ .

5. Figure 5.5 illustrates the two biological reference points  $F_{\max}$  and  $F_{0.1}$ .



**Figure 5.5** Long-term variation of  $Y/R$  and  $B/R$  against  $F$  and points corresponding to  $F_{\max}$  and  $F_{0.1}$

$\bar{B}_{0.1}$  and  $Y_{0.1}$  are the values of  $\bar{B}$  and  $Y$  corresponding to  $F_{0.1}$

$F_{0.1}$  is always smaller than  $F_{\max}$

$\bar{B}_{0.1}$  is always larger than  $\bar{B}$

$Y_{0.1}$  is always smaller than  $Y_{\max}$ , although, in practice, the difference is not large.

The second sentence above indicates the advantages of  $\bar{B}_{0.1}$  over  $B_{\max}$ . The last sentence shows that  $Y_{0.1}$  is not the largest possible catch, but is acceptable as a target point of management. The fact that  $\bar{B}_{0.1}$  is larger than  $\bar{B}_{\max}$  suggests that the fishing level  $F_{0.1}$  is preferable to  $F_{\max}$  as TRP.

Notice that  $F_{0.1}$  can be calculated even when the curve is asymptotical or flat-top.

Another value of  $F_{0.1}$  will be obtained if the exploitation pattern changes.

In the years 1960-70  $F_{0.1}$  started to be preferred to  $F_{max}$  as a target point by resource managers, and it was adopted in the 80's, as a long-term objective by many International Fisheries Commissions and by the EEC.

### 5.2.3 $F_{med}$

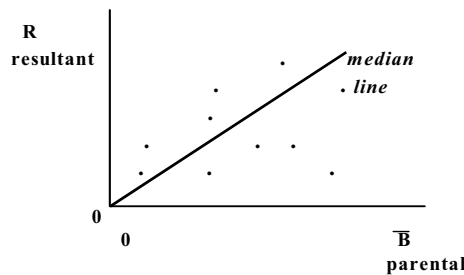
#### 1. *Definition*

This target point considers the relation S-R between the stock and the resulting recruitment, in order to avoid the assumption of a constant recruitment.

Let (the spawning or total) biomasses and the resulting recruitments for each year of a certain period of time be known. In this case, one can calculate the median value of the ratios between the annual spawning biomasses and the corresponding recruitments.

$F_{med}$  is defined as the F value corresponding to the median B/R ratio in the long-term B/R relation against F.

Usually,  $F_{med}$  is illustrated by considering the graph of the points corresponding to pairs of values of parental biomass (total or spawning),  $\bar{B}$ , during that year and the resulting recruitment, R. Figure 5.6 shows this situation.



**Figure 5.6 Illustration of a median line**

The marked line is a line passing through the origin, which separates the total number of points in equal parts, that is, 50 percent of the points are in the upper part and 50 percent are in the lower part of the line. This line is designated as the median line, or 50 percent line, which can be explained as follows: in 50 percent of the years of the considered period the values of R were smaller than the values of R which were estimated by the median line (or, in 50 percent of the years of the referred period the values of R were bigger than the values of R estimated by the median line).

As seen in Section 4.5 the slope (B/R) of each line marked in the graph, is associated with a certain value of the fishing level, F. The value of F associated with the median line is then, the median target point,  $F_{med}$ .

It can be said that, given a certain level of parental biomass and knowing the  $\bar{B}/R$  corresponding to  $F_{med}$ , then there is a 50 percent probability that the resulting recruitment will be less than (or greater than) the value indicated by the median line.

## 2. *Calculation of $F_{med}$*

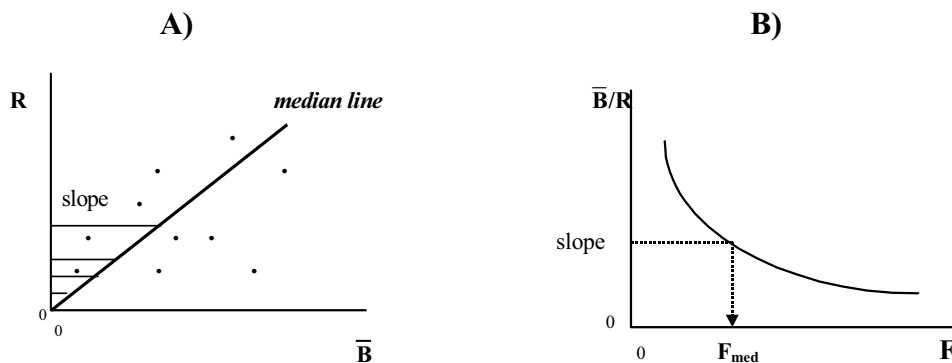
In order to determine the value of  $F_{med}$  it is necessary to consider the long-term relation between the resulting biomass per recruit and the fishing level,  $F$ , (Section 4.4, Figure 4-D).

The determination of  $F_{med}$  can be done mathematically or graphically.

To make the mathematical calculation of  $F_{med}$  the ratio  $\bar{B}/R$  has to be determined for each pair of values ( $B$ ,  $R$ ). Those values have to be ordered and then the median value,  $\bar{B}/R_{median}$  can be calculated.

In the long-term relation of the Biomass per Recruit against  $F$ , the value of  $F_{med}$  is the value of  $F$  that corresponds to the median value previously obtained.

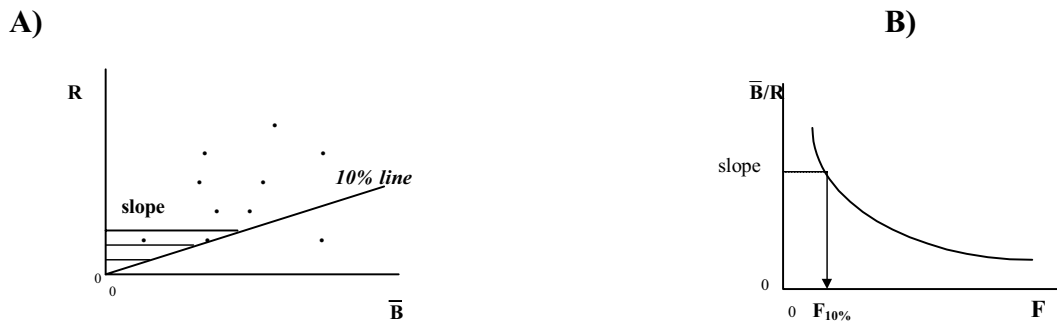
To make the graphical calculation, notice that in Figure 5.6 the slope of the median line against axis  $R$  is equal to  $\bar{B}/R$ . This value will be the basis for the calculation of value  $F_{med}$  in the graph of  $\bar{B}/R$  against  $F$ , in the long-term projection. Figure 5.7(A-B) illustrates the calculation.



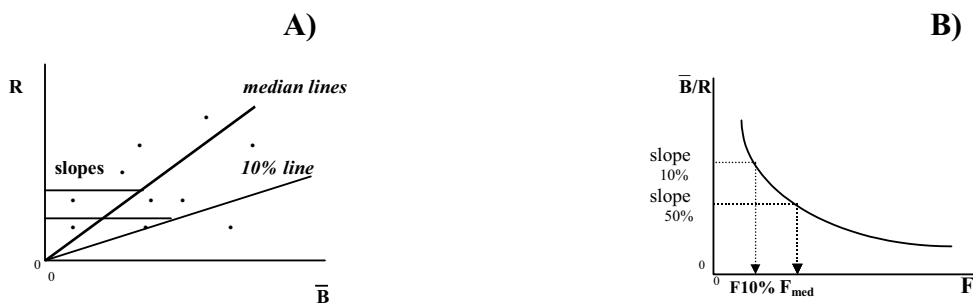
**Figure 5.7** Illustration of the graphical calculation of  $F_{med}$

Other straight lines, corresponding to probabilities other than 50 percent, can be marked. Figure 5.8 illustrates the graphical calculation of  $F_{10\%}$ . The line marked on Figure 5.8A, separates the points, in such a way that 10 percent of them remain below the line (or 90 percent of the points stay above the line). So, this line has been designated as the 10 percent line.

Notice that the slope of the 10 percent line with axis  $R$  is larger than the slope of the median line, as shown in Figure 5.9B. In this way,  $F_{10\%}$ , or  $F_{low}$ , is smaller than  $F_{med}$ .



**Figure 5.8** Illustration of the graphical calculation of  $F_{10\%}$



**Figure 5.9** Graphical illustration of the slopes and corresponding  $F$ 's for the median and 10 percent lines

### Comments

1. The target point  $F_{med}$  intends to ensure an acceptable level of biomass based on the empirical relation S-R.
2. Other percentages can also be adopted, corresponding to straight lines which estimate different probabilities of recruitments which are less than those indicated by the median line. So,  $F_{high}$  would be the fishing level corresponding to the 90 percent line, for which the recruitments of 90 percent of the observed years would be less than those estimated by the line.
3.  $F_{90\%}$ , as will be seen in the following Section, is also considered as a Limit Point (LRP).
4. Notice that the slope of the 10 percent line with axis R is larger than the slope of the median line and therefore,  $F_{10\%}$  (or  $F_{low}$ ) is smaller than  $F_{med}$  (Figure 5.9B).
5.  $F_{med}$  was used in management, in recent years, particularly with Iberic sardines.
6. The biomass used can be the total biomass,  $\bar{B}$ , but is frequently the spawning biomass, SP.
7. If the median line does not pass through a marked point in the scatter plot (which happens when there is an even number of points) then one can use any straight line passing between the two central points, for instance, the mid-line. In any case,  $F_{med}$  is always an approximate value.

## 5.2.4 $F_{MSY}$

### *Definition*

$F_{MSY}$  is defined as being the value of  $F$  which produces the maximum yield in the long-term. It is necessary to select an S-R relation to estimate  $F_{MSY}$ . This point is different from  $F_{max}$ .

## 5.3 BIOLOGICAL LIMIT REFERENCE POINTS

### ( $B_{loss}$ , MBAL, $F_{crash}$ and $F_{loss}$ )

There are several proposals for  $F_{lim}$  and  $B_{lim}$ . For each stock, the adopted values of  $F_{lim}$  and  $B_{lim}$  depend on the characteristics of the stock and on its exploitation. What is important is that the adopted LRP be a value that allows an exploitation level which avoids dangerous situations of stock renewal.

Some of these points are derived from the observed values of Biomass and of Resulting Recruitment. Some examples of this type are  $B_{loss}$  and MBAL. These LRP are also usually classified by some authors as non-parametric, because their determination does not depend on any particular model of the S-R relation.

Another category of LRP points, classified as parametric, is derived from S-R models.

$F_{crash}$  will be mentioned.

Let us also mention the category of LRP points involving observed values and values obtained by the application of S-R models, like, for example,  $F_{loss}$ .

### 5.3.1 $B_{loss}$

$B_{loss}$  is the smallest spawning biomass *observed* in the series of annual values of the spawning biomass (Lowest Observed Spawning Stock).

### 5.3.2 MBAL

More satisfactory is the LRP designated as Minimum Biological Acceptable Level, MBAL. In fact, this LRP is a spawning biomass level below which, observed spawning biomasses over a period of years, are considered unsatisfactory and the associated recruitments are smaller than the mean or median recruitment.

### 5.3.3 $F_{crash}$

The name itself shows that it is a limit that corresponds to a very high value of  $F$ , showing a great probability of collapse of the fishery.

$F_{crash}$  is the fishing level  $F$  which will produce a long-term spawning biomass per recruit (S/R) equal to the inverse of the instantaneous rate of variation of  $R$  with the biomass, at the initial point ( $S = 0$ ,  $R = 0$ ). With the S-R models of Section 4.5 that value is the parameter  $1/\alpha$  of the models.

In order to make the graphical determination of this LRP one can start by obtaining the slope of the angle that the tangent to the S-R curve makes with the  $R$  axis at the origin. Afterwards, and starting from the relation  $\bar{B}/R$  against  $F$ , the value of  $F$  that corresponds to the value

$\bar{B}/R$  indicated by that slope is obtained.  $F_{\text{crash}}$  will then be the value of  $F$  corresponding to  $\bar{B}/R$  equal to that slope, in the long-term relation  $\bar{B}/R$  against  $F$ .

#### 5.3.4 $F_{\text{Loss}}$

$F_{\text{loss}}$  is usually defined as the fishing level  $F$  which will produce a long-term spawning biomass per recruit ( $S/R$ ) associated to  $B_{\text{loss}}$ .

To determine this limit point, first obtain the value of  $R$  corresponding to  $B_{\text{loss}}$  on the adjusted curve  $S-R$ . Then, calculate  $B_{\text{loss}}/R$  and find the value of  $F$ , in the long-term relation  $B/R$  against  $F$ .

Most of the Limit Points shown have been criticised for depending on the observed values or on the adjustment of the  $S-R$  relation.

### 5.4 PRECAUTIONARY REFERENCE POINTS - $F_{\text{pa}}$ , $B_{\text{pa}}$

As previously mentioned, the Precautionary Principle recommends that the assessments should be done even when the basic data presents some gaps. This recommendation implies that, in this case, the determination of the Biological Reference Points will not be very precise. The uncertainties of the estimates should be calculated, and it is necessary to mention the assumptions and models which have been used.

One suggestion to determine  $F_{\text{pa}}$  and  $B_{\text{pa}}$  might be to estimate  $F_{\text{lim}}$  or  $B_{\text{lim}}$  and from these values, to apply the following empirical rules :

$$F_{\text{pa}} = F_{\text{lim}} \cdot e^{-1.645 \cdot \sigma} \quad \text{and} \quad B_{\text{pa}} = B_{\text{lim}} \cdot e^{+1.645 \cdot \sigma}$$

The constant  $\sigma$  is one measure associated with the uncertainty in the estimation of the fishing mortality level,  $F$ . The values obtained in several fisheries indicate that values of  $\sigma$  are within the interval (0.2, 0.3) (ICES, 1997). In practice, it can be said that  $F_{\text{pa}}$  is between  $0.47F_{\text{lim}}$  and  $0.61F_{\text{lim}}$ , and  $B_{\text{pa}}$  is between  $1.39B_{\text{lim}}$  and  $1.64B_{\text{lim}}$ .

It is important to make clear that the target points may also, in certain cases, be considered as limit or precautionary points depending on the combined analyses of the exploitation of the stock and of the biological reference points obtained.



## 5.5 FISHERIES REGULATION MEASURES

The regulation measures aim to control the fishing level and the exploitation pattern applied to the stock for an adequate exploitation.

The most common regulation measures to control fishing levels are:

- Limitation of the number of fishing licences.
- Limitation of the total fishing effort each year (limiting fishing days, number of trips, number of days at sea, etc.).
- Limitation of Total Allowable Catch (TAC)

TAC is a measure that directly controls the catch and, indirectly, the fishing level. It is convenient to combine the TAC with the allocation of quotas of this total TAC for each component of the fleet. In this way, the competition between vessels to fish the maximum possible catch, as quickly as possible, before the TAC is reached, can be avoided.

The system of quotas allocated to each vessel is called Individual Quotas (IQ).

The regulation measures to correct the exploitation pattern are usually called technical measures. Some of these measures are :

- The minimum size (or weight) of the landed individuals.
- The minimum mesh size of the fishing nets.
- The prohibition of fishing in spawning.
- The closed areas and periods for the protection of juveniles.

The fishing management have the duty to promote legislation and the application of the regulation measures. (In the particular case of the EU, and for the stocks of the Economic Exclusive Zones (EEZ's) of the member states, the Commission decides on the measures to be taken). In any case, management needs the analyses on the state of the stock and its exploitation and on the effects of the recommended measures. That study must be done by the fisheries scientists of each country or region and their Fishing Research Institutes, who will have to calculate the projections of the stocks. The International Council for the Exploitation of the Sea (ICES) analyses the assessments and recommends regulation measures and the expected effects of the application of those measures to the Commission, as well as the consequences of their non-application.

The short-term projections, as well as the regulation measures, only make sense if the long-term objectives of fisheries management are previously analysed and defined. Short-term projections of the stock and of its fishing must also be made by the scientists.

### *Comments*

1. Management needs to define the fishing objectives, based on the long-term projections. Those objectives are valid for a period of years, even if they can be adjusted every year.

2. The regulation measures, on the contrary, have to be established every year, although some of them may be valid for more than one year. Some technical measures, like the minimum mesh sizes of the fishing nets or the minimum size of the landed individuals, are valid for several years.
3. All the measures have advantages, difficulties and disadvantages regarding the purposes they intend to reach.

The concession of fishing licences is a common practice almost everywhere, with a limited total number.

TACs and quotas, because they control the catches, have caused misleading declarations about catches.

The direct limitation of the total fishing effort ( $f$ ), is based on the assumption that the measure causes a similar limitation on the fishing mortality coefficient ( $F$ ). However, this relation may not be proportional. In the first place, it is difficult to measure the fishing effort of the different fishing gears and of all the involved fleets and it is also difficult to express it in units that respect the proportionality between  $F$  and  $f$ . Secondly, the capturability of several gears may increase (and consequently increase  $F$ ) without increasing the fishing effort. Finally, the expected proportionality between  $F$  and  $f$  may not be true. In any case, what matters, is not to forget that there is a relation between  $F$  and  $f$ .

4. The protection of the juveniles should be carried out during the whole year and will preferably control the fishing mortality throughout the year. The occasional measures, like areas and periods when fishing is forbidden for the protection of juveniles, require annual investigations in order to discover whether there are exclusive concentrations of juveniles, to assess the effects of that occasional interdiction, and to find out the consequences of the interdiction on other species, etc. The minimum size of the landed individuals, does not mean that smaller individuals are not caught, but only that they are not landed. The difference between the catch and the landing is the so-called rejections to the sea. It is clear that if the individuals are caught and rejected to the sea, the fishing mortality is larger than the one suggested by the landings. The minimum landing size of the individuals may have the effect of dissuading fishermen from catching small individuals. Currently, some countries are forcing the landing of all fish caught.
5. The closed spawning areas and seasons, to save the spawning biomass and indirectly protect recruitment, is far from effective in the latter objective. In fact, large spawning biomasses correspond to a large number of eggs, but that does not necessarily imply bigger recruitments, as seen in Section 4.5. It is also not always true that forbidding fishing during the spawning, and not forbidding before (or after) the spawning, protects the spawning biomass. The only way to protect the spawning biomass will be to control fishing level during the whole year. Finally, it has to be said that, in any case, the interdiction of fishing in the spawning area and period, or on any other occasion, always represents a reduction of the fishing effort. This is not a major inconvenience and in some cases, may even be beneficial.
6. It has to be stressed that no regulation measure will accomplish its objectives without observing two conditions :

The understanding of the fishermen (broadly speaking) that the measure is good for the fishery. Hence, it is important to discuss the scientists' conclusions, their objectives, their reasons and the expected effects.

An efficient fiscalization in the ports and at sea! The 200-mile exclusive zone may be very vast and the fiscalization very expensive, but it is not necessary to fiscalize the whole area intensively. It is enough to control the areas of larger catches more intensively and the remaining areas less so.

During the last few years, new ways of controlling access to fisheries resources and exploitation levels are being implemented. Some examples are the establishment of Individual Transferable Quotas systems (ITQ), co-management systems or even the system of regional or municipal management, where some management responsibilities are attributed to the resource users themselves.

7. The ITQ management system is based on the abusive assumption that only the economically efficient and profitable vessels deserve to be active in fishing. So, TAC's are divided into individual quotas, to be auctioned for the best offer.

The co-management system delegates a great part of the responsibility of management to those who directly exploit the fishing resources - managers, fishermen and their professional organizations or unions. With this system neither are quotas sold in auctions, nor are the fishing licences lost.

These are the most well known systems.

The ITQ system presents the following inconveniences : permanent loss of the titles of quotas and of fishing licences; concentration of quotas in the hands of a small group of people (who may not even belong to the fishing sector or are even foreigners); and underestimation of the social, human and cultural aspects, in favour of economic efficiency criteria.

On the contrary, the co-management system, is concerned with the social aspects of the people involved; it seeks their direct and conscientious co-participation with government authorities in the management responsibilities, including fiscalization.